Math 166 - Week in Review #8

Section 5.1 - Introduction to Matrices

- The **order** (size) of a matrix is always *number of rows* × *number of columns*.
- c_{ij} represents the entry of the matrix C in row i and column j.
- To add and subtract matrices, they must be the same size.
- When adding or subtracting matrices, add or subtract corresponding entries.
- A scalar product is computed by multiplying each entry of a matrix by a scalar (a number).
- Transpose - The rows of the matrix A become the columns of A^T.
- The **zero matrix** of order $m \times n$ is the matrix O with m rows and n columns, all of whose entries are zero.

Section 5.2 - Multiplication of Matrices

- The matrix product AB can be computed only if the number of columns of A equals the number of rows of B.
- If $C = AB$, then c_{ij} is computed by multiplying the i^{th} row of A by the j^{th} column of B.
- Identity Matrix - Denoted by I_n, the identity matrix is the $n \times n$ matrix with 1's down the main diagonal (from upper left corner to lower right corner) and 0's for all other entries.
- If A is $m \times n$, then $AI_n = A$ and $I_m A = A$.
- In general, matrix multiplication is not commutative.

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1. Let $A = \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}$, $\underset{2\times 3}{B} = \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}$, $\underset{3\times 2}{C} = \begin{bmatrix} 4 & -5 \\ 0 & b \\ 7 & -10 \end{bmatrix}$, and $\underset{2\times 3}{D} = \begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}$. Compute each of the following:

(a) $B + 3D$

$$= \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix} + 3\begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix} + \begin{bmatrix} 15 & -3 & 9a \\ 6 & 18 & 12 \end{bmatrix} = \boxed{\begin{bmatrix} 17 & -6 & 9a \\ 14 & k+18 & 6 \end{bmatrix}}$$

(b) $2C + B$

2C+B is not possible since C is 3×2 and B is 2×3. Matrices must be the same size to add.

$$A = \overset{2\times 2}{\begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}}, \quad B = \overset{2\times 3}{\begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}}, \quad C = \overset{3\times 2}{\begin{bmatrix} 4 & -5 \\ 0 & b \\ 7 & -10 \end{bmatrix}}, \text{ and } D = \overset{2\times 3}{\begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}}$$

(c) $4D - 3C^T$

$$= 4\begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix} - 3\begin{bmatrix} 4 & 0 & 7 \\ -5 & b & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & -4 & 12a \\ 8 & 24 & 16 \end{bmatrix} + \begin{bmatrix} -12 & 0 & -21 \\ 15 & -3b & 30 \end{bmatrix} = \boxed{\begin{bmatrix} 8 & -4 & 12a-21 \\ 23 & 24-3b & 46 \end{bmatrix}}$$

(d) $4a_{21} - 2c_{32} + 7d_{13}$

$$= 4(3) - 2(-10) + 7(3a)$$

$$= 12 + 20 + 21a$$

$$= \boxed{32 + 21a}$$

(e) DB

$$\begin{array}{ccc} \underline{\text{Size of } D} & & \underline{\text{Size of } B} \\ 2\times 3 & \neq & 2\times 3 \end{array}$$

DB is not possible since the number of columns of D does not equal

(f) $B^T DA$

$$\begin{array}{ccccc} \underline{\text{Size of } B^T} & & \underline{\text{Size of } D} & & \underline{\text{Size of } A} \\ 3\times 2 & \checkmark = & 2\times 3 & \neq & 2\times 2 \end{array}$$

$B^T DA$ is not possible since the number of columns of D does not equal the number of rows of A.

$$A = \underset{2\times 2}{\begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}},\quad B = \underset{2\times 3}{\begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}},\quad C = \underset{3\times 2}{\begin{bmatrix} 4 & -5 \\ 0 & b \\ 7 & -10 \end{bmatrix}}, \text{ and } D = \underset{2\times 3}{\begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}}$$

(g) CD^T

$$\frac{\text{Size of } C}{3\times 2} \quad \frac{\text{Size of } D^T}{3\times 2} \qquad 2 \neq 3$$

CD^T is not possible since the number of columns of C is not equal to the number of rows of D^T.

(h) BB^T

$$\frac{\text{Size of } B}{2\times 3} \quad \frac{\text{Size of } B^T}{3\times 2} \qquad 3 = 3 \checkmark \qquad 2\times 2$$

$$BB^T = \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}\begin{bmatrix} 2 & 8 \\ -3 & k \\ 0 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(2)+(-3)(-3)+(0)(0) & (2)(8)+(-3)(k)+(0)(-6) \\ (8)(2)+(k)\cdot(-3)+(-6)(0) & (8)(8)+(k)(k)+(-6)(-6) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 13 & 16-3k \\ 16-3k & k^2+100 \end{bmatrix}}$$

(i) $A^2 = AA$

$$\frac{\text{Size of } A}{2\times 2} \quad \frac{\text{Size of } A}{2\times 2} \qquad 2 = 2 \checkmark \qquad 2\times 2$$

$$A^2 = \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}\begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 16+3x & 4x-7x \\ 12-21 & 3x+49 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 16+3x & -3x \\ -9 & 3x+49 \end{bmatrix}}$$

2. Solve for x and y:

$$3\begin{bmatrix} 2 & x \\ 5y & -1 \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ 3y & -5 \end{bmatrix}^T = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 3x \\ 15y & -3 \end{bmatrix} - \begin{bmatrix} -6 & 3y \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

$$\begin{bmatrix} 12 & 3x-3y \\ 15y-1 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

$3x-3y=-7$

$15y-1=-2x$

$3x-3y=-7$

$2x+15y=1$

$$\left[\begin{array}{cc|c} 3 & -3 & -7 \\ 2 & 15 & 1 \end{array}\right] \xrightarrow{rref} \begin{array}{ccc} x & y & \\ \end{array} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 1/3 \end{array}\right]$$

$$\boxed{\begin{aligned} x &= -2 \\ y &= \tfrac{1}{3} \end{aligned}}$$

3. The times (in minutes) required for assembling, testing, and packaging large and small capacity food processors are shown in the following table:

		A Assembling	T Testing	P Packaging
L	Large	45	15	10
S	Small	30	10	5

(a) Define a matrix T that summarizes the above data.

$$T = \begin{array}{c} \\ L \\ S \end{array}\!\!\begin{array}{c} \begin{array}{ccc} A & T & P \end{array} \\ \begin{bmatrix} 45 & 15 & 10 \\ 30 & 10 & 5 \end{bmatrix} \end{array}$$

(b) Let $M = \begin{array}{c} \begin{array}{cc} L & S \end{array} \\ \begin{bmatrix} 100 & 200 \end{bmatrix} \end{array}$ represent the number of large and small food processors ordered, respectively. Find MT and explain the meaning of its entries.

Size of M Size of T

$1\times 2 \quad 2\times 3$

1×3

$$MT = \text{total orders}\begin{bmatrix} 100 & 200 \end{bmatrix}\begin{bmatrix} 45 & 15 & 10 \\ 30 & 10 & 5 \end{bmatrix} = \text{Total}\underbrace{\begin{bmatrix} 10500 & 3500 & 2000 \end{bmatrix}}$$

Total assembling time, testing time, and packaging time for all orders.

(c) If assembling costs \$3 per minute, testing costs \$1 per minute, and packaging costs \$2 per minute, find a matrix C that, when multiplied with T, gives the total cost for making each size of food processor.

Total cost for large: $45*3 + 15*1 + 10*2 = 170$

(row * column)

$$\boxed{\text{Let } C = \begin{array}{c} A \\ T \\ P \end{array}\!\!\overset{\text{Cost/min}}{\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}}.}$$

Answer

Then $TC = \begin{array}{c} L \\ S \end{array}\!\!\begin{bmatrix} 45 & 15 & 10 \\ 30 & 10 & 5 \end{bmatrix} \begin{array}{c} A \\ T \\ P \end{array}\!\!\overset{\text{Cost/min}}{\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}}$

$$TC = \begin{array}{c} L \\ S \end{array}\!\!\overset{\text{Cost}}{\begin{bmatrix} 170 \\ 110 \end{bmatrix}}$$

4. If $A = \begin{bmatrix} 4 & 0 & k \\ -9 & m & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & j & 8 \\ 5 & n & -6 \end{bmatrix}$, and if $C = B^T A$, then find

$B^T = \begin{bmatrix} -3 & 5 \\ j & n \\ 8 & -6 \end{bmatrix}$

Size of B^T Size of A

3×2 ✓ 2×3

(a) c_{32} = row 3 of B^T * col 2 of A

$= [8 \ \ -6] \begin{bmatrix} 0 \\ m \end{bmatrix}$

$= \boxed{-6m}$

(b) c_{13} = row 1 of B^T * col 3 of A

$= [-3 \ \ 5] \begin{bmatrix} k \\ 2 \end{bmatrix}$

$= \boxed{-3k + 10}$

5. Acme Flowers is a florist shop with three locations–one in San Antonio (SA), one in Dallas (D), and one in Houston (H). Each shop makes three standard arrangments A, B, and C. The matrix M below shows the number of each type of arrangement ordered in the month of January. The matrix N below shows the number of roses (R), carnations (C), and chrysanthemums (M) used in each type of arrangment.

$$M = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} SA & D & H \end{array} \\ \begin{bmatrix} 18 & 20 & 16 \\ 12 & 17 & 10 \\ 13 & 11 & 9 \end{bmatrix} \end{array} \qquad N = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} R & C & M \end{array} \\ \begin{bmatrix} 5 & 10 & 2 \\ 7 & 6 & 3 \\ 9 & 12 & 5 \end{bmatrix} \end{array}$$

How should these matrices be multiplied to produce a matrix T that gives the total number of each type of flower needed at each location to meet January's orders?

# of Roses needed in San Antonio $= 18*5 + 12*7 + 13*9 = 291$

$$M^T = \begin{array}{c} \\ SA \\ D \\ H \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 18 & 12 & 13 \\ 20 & 17 & 11 \\ 16 & 10 & 9 \end{bmatrix} \end{array} \qquad N = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} R & C & m \end{array} \\ \begin{bmatrix} 5 & 10 & 2 \\ 7 & 6 & 3 \\ 9 & 12 & 5 \end{bmatrix} \end{array}$$

Answer

$$\boxed{M^T N} = \begin{array}{c} \\ SA \\ D \\ H \end{array} \begin{array}{c} \begin{array}{ccc} R & C & m \end{array} \\ \begin{bmatrix} 291 & 408 & 137 \\ 318 & 434 & 146 \\ 231 & 328 & 107 \end{bmatrix} \end{array}$$

Check: # of mums needed in Dallas $=$

$20*2 + 17*3 + 11*5 = 146$