Math 166 - Week in Review #1

Sections L.1 and L.2 - Statements, Connectives, and Truth Tables

- A **statement** is a declarative sentence that can be classified as either true or false, but not both.
- **Simple statements** are statements expressing a single complete thought.
- We use the lowercase letters p, q, r, etc. to denote simple statements.
- Statements that contain at least one logical connective are called **compound statements**.
- A **conjunction** is a statement of the form "p and q" and is represented symbolically by $p \wedge q$.

  The conjunction $p \wedge q$ is true if *both* p and q are true; it is false otherwise.
- A **disjunction** is a statement of the form "p or q" and is represented symbolically by $p \vee q$.

  The disjunction $p \vee q$ is **false** if *both* p and q are false; it is true in all other cases.
- An ***exclusive* disjunction** is a statement of the form "p or q" and is represented symbolically by $p \veebar q$.

  The disjunction $p \veebar q$ is **false** if *both* p and q are false AND it is **false** if both p and q are true; it is true only when exactly one of p and q is true.
- A negation is a statement of the form "not p" and is represented symbolically by $\sim p$.

  The statement $\sim p$ is true if p is false and vice versa.
- A statement is called a **tautology** if its truth value is always *true*, no matter what the truth values of the simple component statements are.
- A statement is called a **contradiction** if its truth value is always *false*, no matter what the truth values of the simple component statements are.

Section 1.1 - Introduction to Sets

- A *set* is a well-defined collection of objects.
- The objects in a set are called the *elements* (or members) of the set.
- Example of roster notation: $A = \{a, e, i, o, u\}$
- Example of set-builder notation: $B = \{x | x \text{ is a student at Texas A\&M}\}$
- Two sets are equal if and only if they have exactly the same elements.
- If every element of a set A is also an element of a set B, then we say that A is a *subset* of B and write $A \subseteq B$.
- If $A \subseteq B$ but $A \neq B$, then we say A is a proper subset of B and write $A \subset B$.
- The set that contains no elements is called the empty set and is denoted by $\emptyset$. (NOTE: $\{\} = \emptyset, but \{\emptyset\} \neq \emptyset$.)
- The *union* of two sets A and B, written $A \cup B$, is the set of all elements that belong either to A or to B or to both.

- The *intersection* of two sets A and B, written $A \cap B$, is the set of elements that A and B have in common.
- Two sets A and B are said to be **disjoint** if they have no elements in common, i.e., if $A \cap B = \emptyset$.
- If U is a universal set and A is a subset of U, then the set of all elements in U that are not in A is called the *complement* of A and is denoted A^c.
- De Morgan's Laws - Let A and B be sets. Then

  $(A \cup B)^c = A^c \cap B^c$

  $(A \cap B)^c = A^c \cup B^c$

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1. Determine which of the following are statements.

   (a) Do you know when the review starts?

   No - a question

   (b) What a surprise!

   No - an exclamation

   (c) She wore a black suit to the meeting.

   Yes

   (d) The number 4 is an odd number.

   Yes

   (e) $x - 5 = 4$

   No - ambiguous

   (f) Some of guests ate cake.

   Yes.

   (g) Please take off your hat before entering the MSC.

   No - command

2. Write the negation of the following statements.

   (a) Bob will arrive before 8 p.m.

   Bob will not arrive before 8pm.

   (b) All of the pencils have been sharpened.

   Not all of the pencils have been sharpened

   or At least one of the pencils has not been sharpened.

   (c) None of the sodas are cold.

   Some of the sodas are cold.

   or At least one of the sodas is cold.

3. Consider the following statements:

   p: Sally speaks Italian.
   q: Sally speaks French.
   r: Sally lives in Greece.

   (a) Express the compound statement, "Sally speaks Italian and French, but she lives in Greece," symbolically.

   $(p \wedge q) \wedge r$ or $p \wedge q \wedge r$

   (b) Express the compound statement, "Sally lives in Greece, or she does not speak both Italian and French," symbolically.

   $r \vee \sim(p \wedge q)$

   (c) Write the statement $(p \veebar q) \wedge r$ in English.

   Sally speaks either Italian or French (but not both), and she lives in Greece.

   (d) Write the statement $\sim r \wedge \sim (p \vee q)$ in English.

   Sally does not live in Greece, and she does not speak Italian or French.

4. Construct a truth table for each of the following. Also, state whether the given statement is a tautology, a contradiction, or neither.

   (a) $\sim(\sim p \vee \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(\sim p \vee \sim q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

Neither a tautology nor a contradiction.

(b) $(p \vee \sim q) \wedge q$

p	q	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	F
F	F	T	T	F

$\boxed{\text{Neither}}$

(c) $\sim q \wedge \sim (p \vee r)$

p	q	r	$\sim q$	$p \vee r$	$\sim(p \vee r)$	$\sim q \wedge \sim(p \vee r)$
T	T	T	F	T	F	F
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	T	F	F	F	T	F
F	F	T	T	T	F	F
F	F	F	T	F	T	T

$\boxed{\text{Neither}}$

(d) $\sim (p \wedge q) \vee (q \wedge \sim r)$

p	q	r	$\sim r$	$p \wedge q$	$\sim(p \wedge q)$	$q \wedge \sim r$	$\sim(p \wedge q) \vee (q \wedge \sim r)$
T	T	T	F	T	F	F	F
T	T	F	T	T	F	T	T
T	F	T	F	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	F	F	T	F	T
F	T	F	T	F	T	T	T
F	F	T	F	F	T	F	T
F	F	F	T	F	T	F	T

$\boxed{\text{neither}}$

5. Let U be the set of all A&M students. Define D, A, and C as follows:

$D = \{x \in U | x \text{ watches Disney movies}\}$

$A = \{x \in U | x \text{ watches action movies}\}$

$C = \{x \in U | x \text{ watches comedy movies}\}$

(a) Describe each of the following sets in words.

i. $A \cup C$

The set of all A&M students who watch action movies or comedies (or both).

ii. $D \cap C \cap A^c$

The set of all A&M students who watch Disney movies and comedies but (and) not action movies.

iii. $D \cup A \cup C$

The set of all A&M students who watch Disney movies or action movies or comedies.

(The set of all A&M students who watch at least one of Disney, action, or comedy movies)

iv. $C \cap (D \cup A)$

The set of all A&M students who watch comedies, and who watch Disney or action movies.

(b) Write each of the following using set notation.

i. The set of all A&M students who watch comedy movies but not Disney movies.

$C \cap D^c$

ii. The set of all A&M students who watch only comedies of the three types of movies listed.

$C \cap (D \cup A)^c = C \cap D^c \cap A^c$

iii. The set of all A&M students who watch Disney movies or do not watch action movies.

$D \cup A^c$

6. Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,5,10\}$, $B = \{1,3,5,7,9\}$, and $C = \{2,4,6,10\}$. Find each of the following.

(a) $A \cup B$

$A \cup B = \{1, 5, 10, 3, 7, 9\}$

(b) $B \cap C$

$B \cap C = \emptyset$

(c) C^c

$C^c = \{1,3,5,7,8,9\}$

(d) $A \cap (B \cup C)$

$B \cup C = \{1,3,5,7,9,2,4,6,10\}$
$A = \{1,5,10\}$

$\boxed{A \cap (B \cup C) = \{1,5,10\}}$

(e) $(A \cup C)^c \cup B$

$A \cup C = \{1,5,10,2,4,6\}$
$(A \cup C)^c = \{3,7,8,9\}$
$B = \{1,3,5,7,9\}$

$\boxed{(A \cup C)^c \cup B = \{3,7,8,9,1,5\}}$

(f) How many subsets does C have?

$2^4 = 16$ subsets (since C has 4 elements)

(g) How many proper subsets does C have?

$16 - 1 = 15$ proper subsets.

(h) Are A and C disjoint sets?

$A \cap C = \{10\}$ Since $A \cap C \neq \emptyset$, A and C are <u>not</u> disjoint.

(i) Are B and C disjoint sets?

$B \cap C = \emptyset$, so B and C are disjoint.

7. Use set-builder notation to describe the collection of all history majors at Texas A&M University.

$\{x \mid x$ is a history major at Texas A & M University$\}$

8. Write the set $\{x|x$ is a letter in the word ABRACADABRA$\}$ in roster notation.

$\{a, b, r, c, d\}$

9. Let $U = \{a,b,c,d,e,f,g,h,i\}$, $A = \{a,c,h,i\}$, $B = \{b,c,d\}$, $C = \{a,b,c,d,e,i\}$, and $D = \{d,b,c\}$. Use these sets to determine if the following are true or false.

(a) TRUE FALSE $A \subseteq C$ since $h \in A$ and $h \notin C$

(b) TRUE FALSE $B \subset C$ since $B \subseteq C$ and $B \neq C$

(c) TRUE FALSE $D \subset B$ since $D = B$

(d) TRUE FALSE $\emptyset \subseteq A$ since $\emptyset$ is a subset of every set

(e) TRUE FALSE $\{c\} \in A$ since $c \in A$ and $\{c\} \subseteq A$

(f) TRUE FALSE $d \in C$

(g) TRUE FALSE $C \cup C^c = U$

(h) TRUE FALSE $A \cap A^c = 0$ since $A \cap A^c = \emptyset$

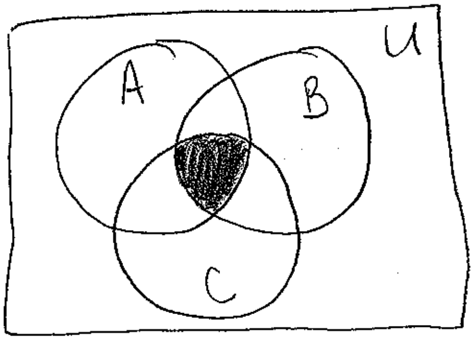
(i) TRUE FALSE $(B \cup B^c)^c = \emptyset$ since $(B \cup B^c)^c = B^c \cap (B^c)^c$

$= B^c \cap B$

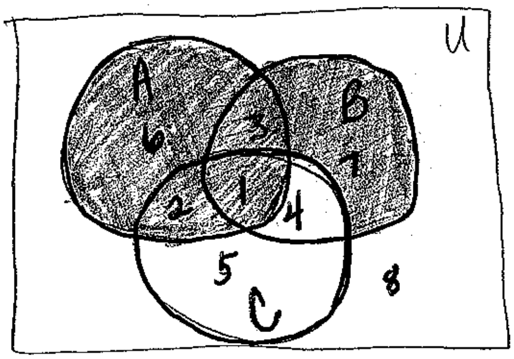
$= \emptyset$

10. Draw a Venn diagram and shade each of the following.

(a) $A \cap B \cap C$



(b) $A \cup (B \cap C^c)$



B: regions 1,3,4,7

C^c: regions 6,3,7,8

$B \cap C^c$: regions 3,7

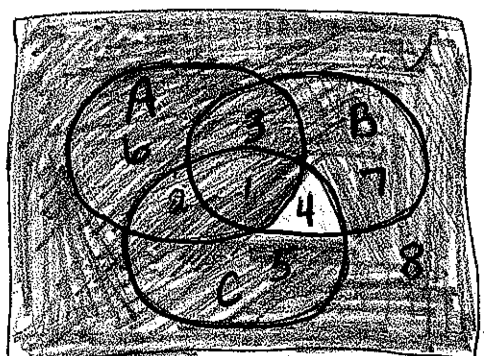
A: regions 6,3,2,1

Shade

$A \cup (B \cap C^c)$: regions $\boxed{3,7,6,2,1}$

In A, or in B but not C

(c) $A \cup (B \cap C)^c$



B - regions 1,3,4,7

C - regions 1,2,4,5

$B \cap C$ - regions 1,4

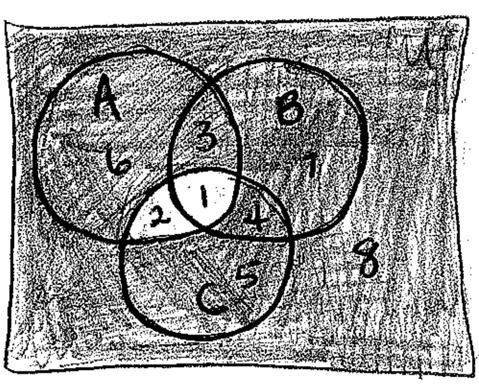
$(B \cap C)^c$ - regions 2,3,5,6,7,8

A - regions 1,2,3,6

$A \cup (B \cap C)^c =$ 2,3,5,6,7,8,1

Shade

(d) $(A \cap C)^c$



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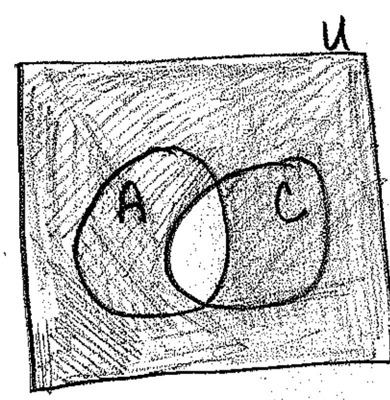

11. Given the following two simple statements, determine the truth values of the compound statements listed below.

p: The planet Mercury is a gas giant.
q: The U.S. is composed of 51 states.

(Note: Both p and q are false.)

(a) $p \wedge \sim q$
$\underbrace{F \wedge T}_{F}$
$\boxed{\text{False}}$

(b) $p \vee q$
$F \vee F$
$\boxed{\text{False}}$

(c) $(\sim p) \underline{\vee} q$
$T \underline{\vee} F$
$\boxed{\text{True}}$

(d) $\sim(\sim p \wedge \sim q)$
$\sim\underbrace{(T \wedge T)}$
$\sim(T)$
$\boxed{\text{False}}$

12. Given the following two simple statements, determine the truth values of the compound statements listed below.

p: *The Grapes of Wrath* was written by John Steinbeck.
q: The Blocker Building is on Texas A&M's West Campus.

(Note: p is true and q is false.)

(a) $p \underline{\vee} (\sim q)$
$T \underline{\vee} T$
$\boxed{\text{False}}$

(b) $p \wedge (\sim q)$
$T \wedge T$
$\boxed{\text{True}}$

(c) $\sim(p \vee q)$
$\sim(T \vee F)$
$\sim(T)$
$\boxed{\text{False}}$

(d) $\sim(\sim p \vee q)$
$\sim((F) \vee F)$
$\sim(F)$
$\boxed{\text{True}}$