Find the following for these functions.
A) Determine the the critical values(cv).
B) Determine the intervals where the function is increasing(inc) and where it is decreasing(dec).
C) Classify the critical values as relative maxima, relative minima or neither.

1. $y=x^{4}+4 x^{3}+4 x^{2}+1$
2. $y=3 x^{5}-5 x^{3}+1$
3. $y=x \ln (x)+1$
4. $y=x^{2} \ln (x)+1$
5. $y=x-4 \ln (3 x-9)$
6. $y=\ln \left(x\left(x^{2}-4\right)^{4}\right)$
7. $y=e^{x^{2}-6 x}$
8. $y=\frac{e^{x^{2}+4 x}}{e^{4 x+5}}$
9. $y=x e^{2 x^{2}-5 x}$
10. $y=\left(x^{2}-9\right)^{2 / 3}$
11. $y=6 x^{2 / 3}-4 x$
12. $y=\sqrt[3]{x^{2}-6 x+8}$

Use the derivative and the domain of $f(x)$ to determine the following for the function $f(x)$.
A) Determine the the critical values(cv).
B) Determine the intervals where the function is increasing(inc) and where it is decreasing(dec).
C) Classify the critical values as relative maxima, relative minima or neither.
13. Domain of $f(x)$ : all real numbers

$$
f^{\prime}(x)=2 x^{2}(x+5)^{3}(x-4)^{4}
$$

14. Domain of $f(x)$ : all real numbers
$f^{\prime}(x)=(4-x)^{3}(x+5)^{2} e^{\left(x^{2}-1\right)}$
15. Domain of $f(x)$ : all real numbers

$$
f^{\prime}(x)=-2\left(x^{2}-16\right)(x+10)(x-15)^{2} e^{4 x^{2}}
$$

16. Domain of $f(x)$ : all real numbers except $x= \pm 2$ $f^{\prime}(x)=\frac{-2\left(x^{2}-5 x+4\right)}{\left(x^{2}-4\right)^{2}}$
17. Domain of $f(x)$ : all real numbers except $x=-5$ $f^{\prime}(x)=\frac{7-3 x}{(x+5)^{3}}$

Here is the graph of $f^{\prime}(x)$. Determine the following for the function $f(x)$.
A) Determine the the critical values(cv).
B) Determine the intervals where the function is increasing(inc) and where it is decreasing(dec).
C) Classify the critical values as relative maxima, relative minima or neither.
18. Domain is all real numbers

19. Domain is all real numbers

20. Domain is all real numbers


## Solutions

1. domain is all real numbers
(a) cv: $x=0,-1,-2$
(b) inc: $(-2,-1),(0, \infty)$
dec: $(-\infty,-2),(-1,0)$
(c) rel max at $x=-1$
rel min at $x=-2$ and $x=0$
2. domain is all real numbers
(a) cv: $x=-1,0,1$
(b) inc: $(-\infty,-1),(1, \infty)$
dec: $(-1,0),(0,1)$
(c) rel max at $x=-1$
rel min at $x=1$
neither at $x=0$
3. domain is all real numbers greater than zero.
(a) cv: $x=e^{-1}$
(b) inc: $\left(e^{-1}, \infty\right)$
dec: $\left(0, e^{-1}\right)$
(c) rel min at $x=e^{-1}$
4. domain is all real numbers greater than zero
(a) cv: $x=e^{-1 / 2}$
(b) inc: $\left(e^{-1 / 2}, \infty\right)$
dec: $\left(0, e^{-1 / 2}\right)$
(c) rel min at $x=e^{-1 / 2}$
5. domain is all real numbers greater than 3
(a) $\mathrm{cv}: x=7$
(b) inc: $(7, \infty)$
dec: $(3,7)$
(c) rel min at $x=7$
6. domain is all real numbers greater than zero and $x \neq 2$
(a) cv: $x=\frac{2}{3}$
(b) inc: $\left(0, \frac{2}{3}\right),(2, \infty)$
dec: $\left(\frac{2}{3}, 2\right)$
(c) rel max at $x=\frac{2}{3}$
7. domain is all real numbers
(a) cv: $x=3$
(b) inc: $(3, \infty)$
dec: $(-\infty, 3)$
(c) rel min at $x=3$
8. domain is all real numbers(hint: simplify before taking the derivative.)
(a) cv: $x=0$
(b) inc: $(0, \infty)$
dec: $(-\infty, 0)$
(c) rel min at $x=0$
9. domain is all real numbers
(a) cv: $x=1, \frac{1}{4}$
(b) inc: $\left(-\infty, \frac{1}{4}\right),(1, \infty)$
dec: $\left(\frac{1}{4}, 1\right)$
(c) rel max at $x=\frac{1}{4}$
rel min at $x=1$
10. domain is all real numbers
(a) cv: $x=-3,0,3$
(b) inc: $(-3,0),(3, \infty)$
dec: $(-\infty,-3),(0,3)$
(c) rel max at $x=0$
rel min at $x=-3$ and $x=3$
11. domain is all real numbers
(a) $\mathrm{cv}: x=0,1$
(b) inc: $(0,1)$
dec: $(-\infty, 0),(1, \infty)$
(c) rel max at $x=1$
rel min at $x=0$
12. domain is all real numbers
(a) $\mathrm{cv}: x=2,3,4$
(b) inc: $(3,4),(4, \infty)$
dec: $(-\infty, 2),(2,3)$
(c) rel min at $x=3$ neither at $x=2$ and $x=4$
13. (a) cv: $x=-5,0,4$
(b) inc: $(-5,0),(0,4),(4, \infty)$
dec: $(-\infty,-5)$
(c) rel min at $x=-5$
neither at $x=0$ and $x=4$
14. (a) cv: $x=-5,4$
(b) inc: $(-\infty,-5),(-5,4)$
dec: $(4, \infty)$
(c) rel max at $x=4$ neither at $x=-5$
15. (a) cv: $x=-10,-4,4,15$
(b) inc: $(-\infty,-10),(-4,4)$ dec: $(-10,-4),(4,15),(15, \infty)$
(c) rel max at $x=-10$ and $x=4$
rel min at $x=-4$
neither at $x=15$
16. (a) cv: $x=1, x=4$
(b) inc: $(1,2),(2,4)$
dec: $(-\infty,-2),(-2,1),(4, \infty)$
(c) rel max at $x=4$
rel min at $x=1$
17. (a) cv: $x=\frac{7}{3}$
(b) inc: $\left(-5, \frac{7}{3}\right)$
dec: $(-\infty,-5),\left(\frac{7}{3}, \infty\right)$
(c) rel max at $x=\frac{7}{3}$
18. (a) cv: $x=-4,-1,4$
(b) inc: $(-4,-1),(4, \infty)$
dec: $(-\infty,-4),(-1,4)$
(c) rel max at $x=-1$
rel min at $x=-4$ and $x=4$
19. (a) cv: $x=-1,2,5$
(b) inc: $(-$ infty, -1$),(5, \infty)$ dec: $(-1,2),(2,5)$
(c) rel max at $x=-1$
rel min at $x=5$
neither at $x=2$
20. (a) cv: $x=-3,1,4,7$
(b) inc: $(-\infty,-3),(1,4),(4,7)$ dec: $(-3,1),(7, \infty)$
(c) rel max at $x=-3, x=7$
rel min at $x=1$
neither at $x=4$
