Find the values of A and B such that these functions will be continuous for all real numbers.

1. $f(x) = \begin{cases} Ax^2 - 3, & \text{if } x \leq 2 \\ Ax + 2, & \text{if } x > 2 \end{cases}$

2. $f(x) = \begin{cases} A^2 x, & \text{if } x < 1 \\ 3Ax - 2, & \text{if } x \geq 1 \end{cases}$

3. $f(x) = \begin{cases} 4x, & \text{if } x \leq -1 \\ Ax + B, & \text{if } -1 < x < 2 \\ -5x, & \text{if } x \geq 2 \end{cases}$

4. $f(x) = \begin{cases} x^2, & \text{if } x < -2 \\ Ax^2 + x + 1, & \text{if } -2 \leq x \leq 2 \\ Bx^2 + 2, & \text{if } x > 2 \end{cases}$

Solutions

We want $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ so that $f(x)$ will be continuous.

1. We want $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ so that $f(x)$ will be continuous.

   $\lim_{x\to 2^-} Ax^2 - 3 = \lim_{x\to 2^+} Ax + 2$
   $4A - 3 = 2A + 2$
   $2A = 5$
   $A = 2.5$

2. We want $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ so that $f(x)$ will be continuous.

   $\lim_{x\to 1^-} A^2 x = \lim_{x\to 1^+} 3Ax - 2$
   $A^2 = 3A - 2$
   $A^2 - 3A + 2 = 0$
   $(A-2)(A-1) = 0$
   $A = 2$ or $A = 1$

3. We want $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$ and $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ so that $f(x)$ will be continuous.

   $\lim_{x\to -1^-} 4x = \lim_{x\to -1^+} Ax + B$
   $-4 = -A + B$

   $\lim_{x\to 2^-} Ax + B = \lim_{x\to 2^+} -5x$
   $2A + B = -10$

   now solve the system of equations to get the solutions. (solve one equation for a letter and substitute into the other equation.)

   Answer: $A = -2$ and $B = -6$

4. We want $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$ and $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ so that $f(x)$ will be continuous.

   $\lim_{x\to -2^-} x^2 = \lim_{x\to -2^+} Ax^2 + x + 1$
   $4 = 4A - 2 + 1$
   $5 = 4A$
   $A = 1.25$

   $\lim_{x\to 2^-} Ax^2 + x + 1 = \lim_{x\to 2^+} Bx^2 + 2$
   $4A + 2 + 1 = 4B + 2$
   $4(1.25) + 2 + 1 - 2 = 4B$
   $4B = 6$
   $B = 1.5$