
The solutions for this quiz MUST be submitted in gradescope by 11:55pm on Thursday, April 18, 2024.

You will be graded on both the correct answer and the correctness of the work that you provide to justify that answer. I expect to see all of your work in a neat and orderly manner.

You are allowed to your your class notes and a calculator when working the quiz. However, I would suggest trying to do quiz without using any other resources to see if you actually know the material. You are not allowed to ask other people for help with the questions. If you need clarification on a question, send me an e-mail or ask during office hours.

You do not need to turn in this cover sheet when you submitt your work. You are expected to tell webassign where on what pages your questions are located.

I would suggest setting an alarm on your phone so that you do not forget to submit the quiz.

You can submitt multiple time to gradescope. Only the last submission is graded.

DO NOT JUST PUT THE QUESTION INTO SYMBOLLAB OR OTHER SIMILAR INTEGRAL CALCULATORS. I expect to see the work done in a clear and concise manner. Correct mathematical notation should be used.

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1. (4 points) Evaluate $\int_C \mathbf{F} \cdot dr$, where $\mathbf{F} =< 2xy^3z^4,\ 3x^2y^2z^4,\ 4x^2y^3z^3 >$ and C is the curve given by

   $x = t$, $y = 2t$ and $z = e^t$ and $0 \leq t \leq 1$.

   Hint: This problem can be done very quickly without using symbolab. Think about the problem.

2. (1 point) A circular path C of radius 3 is parameterized by the formulas $x = 3\cos\theta$ and $y = 3\sin\theta$ for $0 \leq \theta \leq 2\pi$. Green's Theorem was used to convert the line integral $\oint_C Pdx + Qdy$ to the following double integral which was in turn converted to this polar integral.

   $$\int_{\theta=0}^{2\pi} \int_{r=0}^{3} f(r,\theta) r dr d\theta = ... = 15$$

   Give the value of $\oint_C Pdx + Qdy =$______

3. (5 points) Use Green's Theorem to evaluate the line integral along the path C is the trianglar path from $(0,0)$ to $(2,0)$ to $(2,1)$ to $(0,0)$.

   $$\int_C xy\ dx + y^5 dy$$

1] $\text{curl } F = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$

$\text{curl } F = \langle 12x^2y^2z^3 - 12x^2y^2z^3, 8xy^3z^3 - 8xy^3z^3, 6xy^2z^4 - 6xy^2z^4 \rangle$

$= \langle 0, 0, 0 \rangle$

Thus F is conservative

$f_x = 2xy^3z^4 \rightarrow f(x,y,z) = x^2y^3z^4 + c(y,z)$

$f_y = 3x^2y^2z^4 \rightarrow f(x,y,z) = x^2y^3z^4 + c(x,z)$

$f_z = 4x^2y^3z^3 \rightarrow f(x,y,z) = x^2y^3z^4 + c(x,y)$

potential function $f(x,y,z) = x^2y^3z^4$

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at $t=0$ $\quad x=0$, $y=0$, $z=e^0=1$ $\qquad t=1$ $\quad x=1$, $y=2$, $z=e^1=e$

$$\int_C F \cdot dr = f(1,2,e) - f(0,0,1)$$

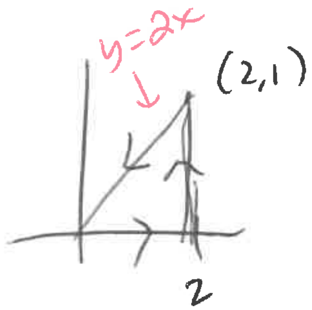
$$= 1^2(2)^3(e)^4 - 0$$

$$= 8e^4$$

Fundamental Thm of Line Integrals

2] The path given is counterclockwise so the final answer is the number 15.

$$\oint_C P\,dx + Q\,dy = 15$$

3] 

$P = xy \qquad Q = y^5$

$P_y = x \qquad Q_x = 0$

$Q_x - P_y = 0 - x = -x$

$$\int_C xy\,dx + y^5\,dy = \iint_D -x\,dA = \int_{x=0}^{2}\int_{y=0}^{\frac{1}{2}x} -x\,dy\,dx$$

$$= \int_{x=0}^{2} -xy\Big|_{y=0}^{\frac{1}{2}x} dx = \int_{x=0}^{2} \frac{-x^2}{2}\,dx = \frac{-x^3}{6}\Big|_0^2$$

$$= -\frac{8}{6} - 0 = -\frac{8}{6} = -\frac{4}{3}$$