Quiz #10(Last Quiz) Key Math 251 April 25, 2024

The solutions for this quiz MUST be submitted in gradescope by 11:55pm on Thursday, April 25, 2024.

You will be graded on both the correct answer and the correctness of the work that you provide to justify that answer. I expect to see all of your work in a neat and orderly manner.

You are allowed to your your class notes and a calculator when working the quiz. However, I would suggest trying to do quiz without using any other resources to see if you actually know the material. You are not allowed to ask other people for help with the questions. If you need clarification on a question, send me an e-mail or ask during office hours.

You do not need to turn in this cover sheet when you submitt your work. You are expected to tell webassign where on what pages your questions are located.

I would suggest setting an alarm on your phone so that you do not forget to submit the quiz.

You can submitt multiple time to gradescope. Only the last submission is graded.

DO NOT JUST PUT THE QUESTION INTO SYMBOLLAB OR OTHER SIMILAR INTEGRAL CALCULATORS. I expect to see the work done in a clear and concise manner. Correct mathematical notation should be used.

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1. Given the vector fields $\mathbf{F} = \langle 1, 3x, 2y \rangle$ and $\mathbf{G} = \langle x, -y, z \rangle$, compute the following.

   (a) $\mathbf{div}(\mathbf{F} \times \mathbf{G})$.

   (b) $\mathbf{curl}(\mathbf{F} + \mathbf{G})$.

2. Compute $\iint\limits_S x + y + z \, dS$ where S is the portion of the plane $2x + 2y + z = 4$ in the first octant.

1] A]

$$F \times G = \begin{vmatrix} i & j & k \\ 1 & 3x & 2y \\ x & -y & z \end{vmatrix} = \langle 3xz + 2y^2, -z + 2xy, -y - 3x^2 \rangle$$

$$\operatorname{div}(F \times G) = 3z + 2x + 0$$

B]

$$F + G = \langle x+1, 3x - y, 2y + z \rangle$$

$$\text{Curl} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle$$

$$= \langle 2 - 0, 0 - 0, 3 - 0 \rangle$$

$$= \langle 2, 0, 3 \rangle$$

2) Surface

$x = x$

$y = y$

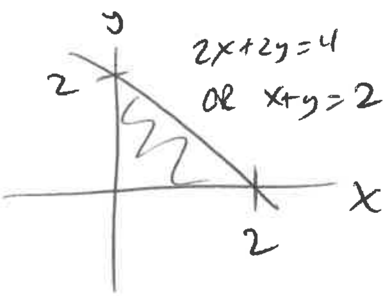
$z = 4 - 2x - 2y$

cross product

$r_x \times r_y = \langle 2, 2, 1 \rangle$

$|r_x \times r_y| = \sqrt{4+4+1} = \sqrt{9} = 3$

$$\iint_S x+y+z \, dS = \iint_D (x+y+4-2x-2y)\, 3\, dA = \int_{x=0}^{2}\int_{y=0}^{2-x} (4-x-y)\, 3 \, dy\, dx$$



$$= 3\int_{x=0}^{2} 4y - xy - \frac{y^2}{2} \Big|_{y=0}^{2-x} dx$$

$$= 3\int_{x=0}^{2} 4(2-x) - x(2-x) - \frac{1}{2}(2-x)^2 \, dx$$

$$= 3\int_{x=0}^{2} 8 - 4x - 2x + x^2 - \frac{1}{2}(4 - 4x + x^2)\, dx$$

$$= 3\int_{x=0}^{2} 8 - 6x + x^2 - 2 + 2x - \frac{x^2}{2}\, dx = 3\int_{x=0}^{2} 6 - 4x + \frac{1x^2}{2}\, dx$$

$$= 3\left[6x - 2x^2 + \frac{1}{6}x^3\right]_0^2 = 3\left(12 - 8 + \frac{8}{6}\right) = 3\left(4 + \frac{4}{3}\right)$$

$$= 12 + 4 = 16$$