Math 251 | Exam 2 B | Thursday, February 29, 2024

Printed Name: ______________________

Section: ______________________

Key

UIN: ______________________

Signature: ______________________

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Section 507: class time TR 12:45
Section 508: class time TR 2:20

- **Show all appropriate work to receive full credit.**
- **<u>You will be graded not merely on your final answer, but also on the quality and correctness of the work leading up to it.</u>**
- If you need more space to work a problem, you may use the back of the cover page or the back of the exam. Please indicate where the problem is located.
- **Calculators are not allowed.**
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Good Luck!

1. (10 points) True or False. Circle your answer.

T F The level curve to the surface $f(x, y) = \dfrac{2x}{x^2 + y^2}$ for $k = 1$ is a circle with center $(-1, 0)$.

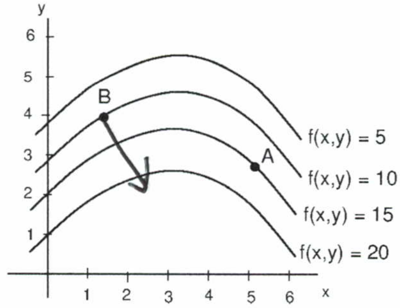
T F IF $f(x, y)$ has a local minimum at (a, b) and f is differentialble at (a, b), then $\nabla f(a, b) = \langle 0, 0 \rangle$.

T F There is a level curve for the function $f(x, y) = 2x^3 + y$ that will contain both of the points $(1, 10)$ and $(2, -3)$ in its graph.

T F If $f(1, 2) = 10$, $f_x(1, 2) = 3$, $f_y(1, 2) = 2$, $f_{xx}(1, 2) = 4$, and $f_{yy}(1, 2) = 5$, then the tangent plane approximation(linearization) of $f(x, y)$ at the point $(1, 2)$ is $L(x, y) = 4x + 5y - 4$.

T F There exists a function f whose partial derivatives are $f_x(x, y) = 3e^{3x+y^2}$ and $f_y(x, y) = 2ye^{3x+y^2}$.

2. (3 points) The following is the graph of level curves for the surface $f(x, y)$. At point **B**, sketch a vector that would represent ∇f.



3. (10 points) Determine whether the critical points given in the table are **local maxima, local minima, or saddle point**. If more information is needed to make a conclusion, then answer **more info needed** in the conclusion blank.

critical point (a,b)	$f_{xx}(a,b)$	$f_{yy}(a,b)$	$f_{xy}(a,b)$	conclusion
$(3,12)$	3	-4	5	$D<0$ Saddle point.
$(8,3)$	5	4	4	$D>0$ Local min.
$(11,7)$	12	3	6	$D=0$ more Info needed.
$(6,8)$	-1	-8	2	$D>0$ Local max
$(6,4)$	-3	-5	-4	$D<0$ saddle point.

$9-x^2-y^2>0$ $\quad$ $xy\geq 0$

$x^2+y^2<9$

4. (4 points) Describe the domain of $f(x,y)=\ln(9-x^2-y^2)+\sqrt{xy}$. Circle the correct letter.

(a) The set of all points (x,y) that lie strictly inside the circle $x^2+y^2=9$ in the quadrants I and III only, including the x and y axis.

(b) The set of all points (x,y) that lie strictly inside the circle $x^2+y^2=9$ in the quadrants I and III only, excluding the x and y axis.

(c) The set of all points (x,y) that lie inside and on the circle $x^2+y^2=9$ in the quadrants I and III only, including the x and y axis.

(d) The set of all points (x,y) that lie inside and on the circle $x^2+y^2=9$ in the quadrants I and III only, excluding the x and y axis.

(e) None of the statements above correctly describe the domain.

5. (5 points) The radius of a right circular cone is decreasing at a rate of $\frac{1}{4}$ in/sec while the height is increasing at a rate of $\frac{4}{3}$ in/sec. At what rate is the volume of the cone changing when the radius is 3 inches and the height is 6 inches?

You may use the fomula $V = r^2h^3$ for some partial credit if you have forgotten the actual volume formula.

$V = \frac{1}{3}\pi r^2 h$ $\qquad r=3 \quad h=6 \quad \frac{dr}{dt} = \frac{-1}{4} \quad \frac{dh}{dt} = \frac{4}{3}$

$$\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt} + \frac{1}{3}\pi r^2 \frac{dh}{dt}$$

$$= \frac{2}{3}\pi (3)(6)\left(\frac{-1}{4}\right) + \frac{1}{3}\pi(9)\left(\frac{4}{3}\right)$$

$$= -3\pi + 4\pi$$

$$= \pi \text{ in}^3/\text{sec}$$

6. (6 points) Assume that you made the following calculation: $\nabla f(4, 6, 1) = \langle 10, 3, 2\rangle$

(a) Find the maximum rate of change of $f(x, y, z)$ at the point (4, 6, 1).

$$|\nabla f| = \sqrt{100+9+4} = \sqrt{113}$$

(b) Find the direction in which $f(x, y, z)$ decreases fastest at the point ((4, 6, 1).

$$-\nabla f = \langle -10, -3, -2\rangle$$

7. (7 points) Find parametric equations of the normal line to the given surface at the specified point.

$x^3 - 5y^2 + z^2 = 21 - 3yz$, $(2, 1, 3)$

$\underbrace{x^3 - 5y^2 + z^2 + 3yz}_{F(x,y,z)} = 21$

$n = \nabla F = \langle 3x^2, -10y + 3z, 2z + 3y \rangle$

$n = \langle 12, -10+9, 6+3 \rangle$
$= \langle 12, -1, 9 \rangle$

$x = 2 + 12t$
$y = 1 - t$
$z = 3 + 9t$

8. (10 points) Given $g(x, y) = 4y^2 - 2y + x^2y$.

(a) Find the gradient at the point $P(1, 3)$.

$\nabla g = \langle 2xy, 8y - 2 + x^2 \rangle$

$\nabla g(1,3) = \langle 6, 24 - 2 + 1 \rangle$
$= \langle 6, 23 \rangle$

(b) Find the rate of change of g at the point $P(1, 3)$ in the direction of the point $\overset{Q}{(3, 4)}$.

$\vec{PQ} = \langle 2, 1 \rangle$

$|\vec{PQ}| = \sqrt{4+1} = \sqrt{5}$

$u = \frac{1}{\sqrt{5}} \langle 2, 1 \rangle$

$D_u g(1,3) = \langle 6, 23 \rangle \cdot \frac{1}{\sqrt{5}} \langle 2, 1 \rangle$
$= \frac{1}{\sqrt{5}} [12 + 23]$
$= \frac{35}{\sqrt{5}}$

9. (6 points) Find the equation of the tangent plane to the surface $z = x^5 + y^3$ at the point (1, 5, 126).

$n = \langle -z_x, -z_y, 1 \rangle = \langle -5x^4, -3y^2, 1 \rangle$

$n = \langle -5, -75, 1 \rangle$

$$-5(x-1) - 75(y-5) + (z-126) = 0$$

10. (8 points) Given $f(x, y) = \ln(x^2 - 4y)$, use differentials(or linearization) to approximate $f(6.4,\ 7.7)$ knowing that $f(6, 8) = \ln(4)$.

$f_x = \dfrac{2x}{x^2-4y}$

$f_x(6,8) = \dfrac{12}{36-32} = \dfrac{12}{4} = 3$

$f_y = \dfrac{-4}{x^2-4y}$

$f_y(6,8) = \dfrac{-4}{4} = -1$

$dx = \Delta x = 6.4 - 6 = .4$

$dy = \Delta y = 7.7 - 8 = -.3$

$f(6.4, 7.7) \approx f(6,4) + f_x\,dx + f_y\,dy$

$= \ln(4) + 3(.4) + (-1)(-.3)$

$= \ln(4) + 1.2 + .3$

$= \ln(4) + 1.5$

11. (6 points) Find the critical points to $f(x, y)$, You **do not** have to determine whether they are maxima, minima, or saddle points.

$$f(x, y) = x^3 + xy^2 + 9x^2 + 3y^2 + 24x$$

$f_x = 3x^2 + y^2 + 18x + 24$

$f_y = 2xy + 6y$

$0 = 2y(x+3)$

$y = 0 \qquad x = -3$

$\underline{y=0)}$

$0 = 3x^2 + 18x + 24$

$0 = 3(x^2 + 6x + 8)$

$0 = 3(x+4)(x+2)$

$x = -4 \qquad x = -2$

$(-4, 0)$

$(-2, 0)$

$\underline{x=-3)}$

$0 = 27 + y^2 - 54 + 24$

$0 = y^2 + 51 - 54$

$0 = y^2 - 3$

$y^2 = 3$

$y = \pm\sqrt{3}$

$(-3, \sqrt{3})$

$(-3, -\sqrt{3})$

12. (7 points) Find z_x for $\qquad x^3z^5 + x^6y^3 = 5 + e^{7yz}$

$$\underbrace{x^3z^5 + x^6y^3 - e^{7yz}}_{F(x,y,z)} - 5 = 0$$

$$z_x = \frac{-F_x}{F_y} = \frac{-(3x^2z^5 + 6x^5y^3)}{5x^3z^4 - 7ye^{7yz}}$$

13. (8 points) Use the method of Lagrange multipliers to find the point on the plane so that the function $f(x, y, z)$ has the maximum value. assume that $x > 0$, $y > 0$, and $z > 0$.

$f(x, y, z) = xyz$ $\qquad\qquad$ $\underbrace{5x + y + 10z}_{g(x,y,z)} = 30$

$f_x = \lambda g_x$ $\qquad$ $f_y = \lambda g_y$ $\qquad$ $f_z = \lambda g_z$

$yz = \lambda 5$ $\qquad$ $xz = \lambda \cdot 1$ $\qquad$ $xy = 10\lambda$

$\frac{yz}{5} = \lambda$ $\qquad$ $xz = \lambda$ $\qquad$ $\frac{xy}{10} = \lambda$

$\frac{yz}{5} = xz$

$yz = 5xz$

$yz - 5xz = 0$

$z(y - 5x)$

$z = 0$ $\qquad$ $y = 5x$

X

$\frac{yz}{5} = \frac{xy}{10}$

$10yz = 5xy$

$10yz - 5xy = 0$

$5y(2z - x) = 0$

$y = 0$ $\qquad$ $2z = x$

X $\qquad$ $z = \frac{x}{2}$

$5x + 5x + 5x = 30$

$15x = 30$

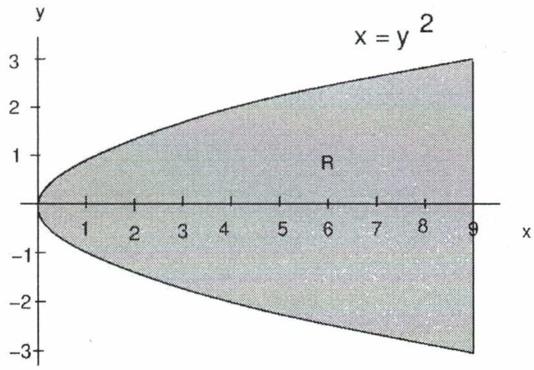
$x = 2$

$y = 10$

$z = 1$

14. (10 points) Find the absolute extrema for $f(x, y)$ on the region R as shown in the picture. Place all points used to calculate the absolute max and min in the table on the right.

$f(x, y) = 7x + 2xy - y^2$



corner points

testing points	function value
(9, 3)	108 Abs max
(9, -3)	0
(0,0)	0
(4, -2)	8

Abs min

$f_x = 7 + 2y$

$0 = 7 + 2y$

$2y = -7$

$y = -\frac{7}{2}$

$f_y = 2x - 2y$

$0 = 2x - 2(-\frac{7}{2})$

$0 = 2x + 7$

$-\frac{7}{2} = x$

not on Region

$x = y^2$

$f = 7y^2 + 2y^3 - y^2$

$= 2y^3 + 6y^2$

$f' = 6y^2 + 12y$

$0 = 6y(y+2)$

$y = 0 \quad y = -2$

(0,0)

(4,-2)

$x = 9$

$f = 63 + 18y - y^2$

$f' = 18 - 2y$

$y = 9$

point (9,9)

not on Region