Math 251 | Exam 2 A | Thursday, February 29, 2024

Printed Name: ______________________

Section: Key

UIN: ______________________

Signature: ______________________ On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

Section 507: class time TR 12:45
Section 508: class time TR 2:20

- **Show all appropriate work to receive full credit.**
- **You will be graded not merely on your final answer, but also on the quality and correctness of the work leading up to it.**
- If you need more space to work a problem, you may use the back of the cover page or the back of the exam. Please indicate where the problem is located.
- **Calculators are not allowed.**
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Good Luck!

1. (10 points) True or False. Circle your answer.

T **F** If $f(1,2) = 10$, $f_x(1,2) = 3$, $f_y(1,2) = 2$, $f_{xx}(1,2) = 4$, and $f_{yy}(1,2) = 5$, then the tangent plane approximation(linearization) of $f(x,y)$ at the point $(1,2)$ is $L(x,y) = 4x + 5y - 4$.

$10 + 3(x-1) + 2(y-2)$

$10 + 3x - 3 + 2y - 4 = 3 + 3x + 2y$

T **F** There is a level curve for the function $f(x,y) = 2x^3 + y$ that will contain both of the points $(1,10)$ and $(2,-3)$ in its graph.

$f(1,10) = 2+10$
$= 12$

$f(2,-3) = 16 - 3 = 13$

T F There exists a function f whose partial derivatives are $f_x(x,y) = 3e^{3x+y^2}$ and $f_y(x,y) = 2ye^{3x+y^2}$.

$f(x,y) = e^{3x+y^2}$

T **F** The level curve to the surface $f(x,y) = \dfrac{2x}{x^2+y^2}$ for $k = 1$ is a circle with center $(-1,0)$.

$1 = \dfrac{2x}{x^2+y^2}$

$x^2+y^2 = 2x$

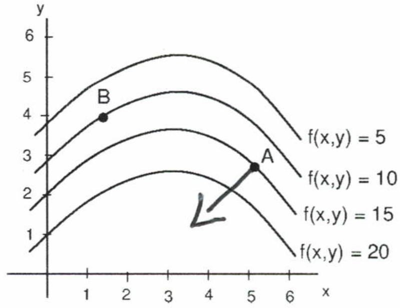
$x^2 - 2x + y^2 = 0$

$(x-1)^2 + y^2 = 1$

center $(1,0)$

T F IF $f(x,y)$ has a local minimum at (a,b) and f is differentialble at (a,b), then $\nabla f(a,b) = \langle 0,0 \rangle$.

2. (3 points) The following is the graph of level curves for the surface $f(x,y)$. At point **A**, sketch a vector that would represent ∇f.



3. (10 points) Determine whether the critical points given in the table are **local maxima, local minima, or saddle point**. If more information is needed to make a conclusion, then answer **more info needed** in the conclusion blank.

critical point (a, b)	$f_{xx}(a, b)$	$f_{yy}(a, b)$	$f_{xy}(a, b)$	conclusion
$(5, 9)$	-1	-8	2	$D>0$ local max
$(5, 5)$	-3	-5	-4	$D<0$ saddle
$(2, 13)$	3	-4	5	$D<0$ saddle.
$(7, 4)$	5	4	4	$D>0$ Local min
$(10, 8)$	12	3	6	$D=0$ more Info needed.

$9-x^2-y^2>0$ $\quad xy\geq 0$

$9>x^2+y^2$

4. (4 points) Describe the domain of $f(x, y) = \ln(9 - x^2 - y^2) + \sqrt{xy}$. Circle the correct letter.

   (a) The set of all points (x, y) that lie inside and on the circle $x^2 + y^2 = 9$ in the quadrants I and III only, excluding the x and y axis.

   (b) The set of all points (x, y) that lie inside and on the circle $x^2 + y^2 = 9$ in the quadrants I and III only, including the x and y axis.

   (c) The set of all points (x, y) that lie strictly inside the circle $x^2 + y^2 = 9$ in the quadrants I and III only, excluding the x and y axis.

   (d) The set of all points (x, y) that lie strictly inside the circle $x^2 + y^2 = 9$ in the quadrants I and III only, including the x and y axis.

   (e) None of the statements above correctly describe the domain.

5. (5 points) The radius of a right circular cone is increasing at a rate of $\frac{1}{2}$ in/sec while the height is decreasing at a rate of $\frac{3}{2}$ in/sec. At what rate is the volume of the cone changing when the radius is 4 inches and the height is 3 inches?

You may use the fomula $V = r^2h^3$ for some partial credit if you have forgotten the actual volume formula.

$r=4 \quad h=3 \quad \frac{dr}{dt}=\frac{1}{2} \quad \frac{dh}{dt}=-\frac{3}{2}$

$V=\frac{1}{3}\pi r^2 h$

$\frac{dV}{dt}=\frac{2}{3}\pi r h\frac{dr}{dt}+\frac{1}{3}\pi r^2\frac{dh}{dt}$

$=\frac{2}{3}\pi(4)(3)(\frac{1}{2})+\frac{1}{3}\pi(16)(\frac{-3}{2})$

$=4\pi-8\pi$

$=-4\pi$ in^3/sec

6. (6 points) Assume that you made the following calculation: $\nabla f(2,1,12)=\langle 1,4,2\rangle$

(a) Find the maximum rate of change of $f(x,y,z)$ at the point (2, 1, 12).

$|\nabla f|=\sqrt{1+16+4}=\sqrt{21}$

(b) Find the direction in which $f(x,y,z)$ decreases fastest at the point (2, 1, 12).

$-\langle 1,4,2\rangle=\langle -1,-4,-2\rangle$

7. (7 points) Find parametric equations of the normal line to the given surface at the specified point.

$x^3 - 5y^2 + z^2 = 21 - 3yz$, $(2, 1, 3)$

$\underbrace{x^3 - 5y^2 + z^2 + 3yz}_{F(x,y,z)} = 21$

$\nabla F = \langle 3x^2, -10y + 3z, 2z + 3y \rangle$

$\nabla F(2,1,3) = \langle 12, -10 + 9, 6 + 3 \rangle$

$= \langle 12, -1, 9 \rangle$

$x = 2 + 12t$

$y = 1 - t$

$z = 3 + 9t$

8. (10 points) Given $g(x, y) = 5x^2 - 3x + xy^2$.

(a) Find the gradient at the point $P(1, 2)$.

$\nabla g = \langle 10x - 3 + y^2, 2xy \rangle$

$\nabla g(1,2) = \langle 10 - 3 + 4, 4 \rangle$

$= \langle 11, 4 \rangle$

(b) Find the rate of change of g at the point $P(1, 2)$ in the direction of the point $\overset{Q}{(4, 1)}$.

$\vec{PQ} = \langle 4-1, 1-2 \rangle = \langle 3, -1 \rangle$

$|\vec{PQ}| = \sqrt{9+1} = \sqrt{10}$

$D_u g(1,2) = \langle 11, 4 \rangle \cdot \frac{1}{\sqrt{10}} \langle 3, -1 \rangle$

$= \frac{1}{\sqrt{10}} [11(3) + 4(-1)]$

$= \frac{1}{\sqrt{10}} (33 - 4) = \frac{29}{\sqrt{10}}$

9. (6 points) Find the equation of the tangent plane to the surface $z = 5x^2 + y^4$ at the point $(2, 1, 21)$.

$z_x = 10x$

$z_y = 4y^3$

$n = \langle -z_x, -z_y, 1 \rangle$

$n = \langle -20, -4, 1 \rangle$

$-20(x-2) - 4(y-1) + (z-21) = 0$

10. (8 points) Given $f(x, y) = \ln(x^2 - 2y)$, use differentials(or linearization) to approximate $f(4.6, 6.5)$ knowing that $f(4, 7) = \ln(2)$.

$f_x = \dfrac{2x}{x^2 - 2y}$

$f_x(4,7) = \dfrac{8}{16-14}$

$= \dfrac{8}{2} = 4$

$f_y = \dfrac{-2}{x^2 - 2y}$

$f_y = \dfrac{-2}{2} = -1$

$dx = \Delta x = 4.6 - 4 = .6$

$dy = \Delta y = 6.5 - 7 = -.5$

$f(4.6, 6.5) \approx f(4,7) + f_x(4,7)\,dx + f_y(4,7)\,dy$

$= \ln(2) + 4(.6) + (-1)(-.5)$

$= \ln(2) + 2.4 + .5$

$= \ln(2) + 2.9$

11. (6 points) Find the critical points to $f(x,y)$, You **do not** have to determine whether they are maxima, minima, or saddle points.

$f(x,y) = 2x^3 + xy^2 + 6x^2 + 2y^2 - 18x$

$f_x = 6x^2 + y^2 + 12x - 18$

$f_y = 2xy + 4y$

$0 = 2y(x+2)$

$y = 0 \quad x = -2$

$\boxed{y=0}$

$0 = 6x^2 + 12x - 18$

$0 = 6(x^2 + 2x - 3)$ $(-3, 0)$

$0 = 6(x+3)(x-1)$ $(1, 0)$

$x = -3, x = 1$

$\boxed{x=-2}$ $0 = 24 + y^2 - 24 - 18$ $(-2, \sqrt{18})$

$18 = y^2$ $(-2, -\sqrt{18})$

$y = \pm\sqrt{18}$

12. (7 points) Find z_y for $x^7y^4 + y^3z^8 = 10 + e^{5xz}$

$\underbrace{x^7y^4 + y^3z^8 - 10 - e^{5xz}}_{F(x,y,z)} = 0$

$$z_y = \frac{-F_y}{F_z} = \frac{-(4x^7y^3 + 3y^2z^8)}{8y^3z^7 - 5xe^{5xz}}$$

13. (8 points) Use the method of Lagrange multipliers to find the point on the plane so that the function $f(x, y, z)$ has the maximum value. assume that $x > 0$, $y > 0$, and $z > 0$.

$f(x, y, z) = xyz$ $\qquad$ $\underbrace{2x + 5y + z}_{g(x,y,z)} = 60$

$f_x = \lambda g_x$ $\qquad$ $f_y = \lambda g_y$ $\qquad$ $f_z = \lambda g_z$

$yz = 2\lambda$ $\qquad$ $xz = 5\lambda$ $\qquad$ $xy = \lambda$

$\frac{yz}{2} = \frac{xz}{5}$

$5yz = 2xz$

$5yz - 2xz = 0$

$z(5y - 2x) = 0$

$\cancel{z = 0}$ $\quad$ $5y = 2x$

$y = \frac{2x}{5}$

$\frac{yz}{2} = xy$

$yz = 2xy$

$yz - 2xy = 0$

$y(z - 2x) = 0$

$\cancel{y = 0}$ $\quad$ $z - 2x = 0$

$z = 2x$

$2x + 5\left(\frac{2x}{5}\right) + 2x = 60$

$6x = 60$

$x = 10$

$y = \frac{20}{5} = 4$

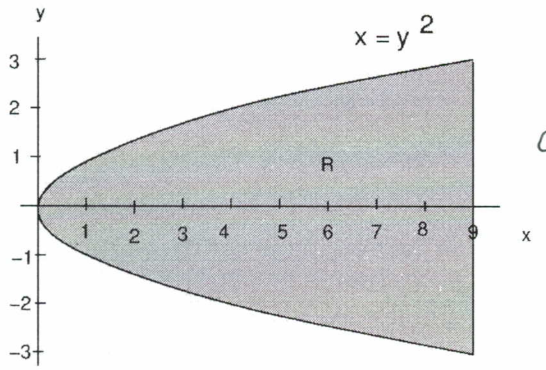
$z = 20$

$\begin{array}{ccc} x & y & z \end{array}$

$(10, 4, 20)$

14. (10 points) Find the absolute extrema for $f(x, y)$ on the region R as shown in the picture. Place <u>all</u> points used to calculate the absolute max and min in the table on the right.

$f(x, y) = 15x - 4xy - 3y^2$



testing points	function value	
$(9,3)$	0	Abs min
$(9,-3)$	216	Abs max
$(0,0)$	0	Abs min
$(4,2)$	16	

corner points

Abs max 216

Abs min 0

$f_x = 15 - 4y$

$0 = 15 - 4y$

$4y = 15$

$y = \frac{15}{4}$

$f_y = -4x - 6y$

$0 = -4x - 6\left(\frac{15}{4}\right)$

$\frac{90}{4} = -4x$

$x = -\frac{90}{16}$

$\left(\frac{-90}{16}, \frac{15}{4}\right)$ not in Region

$\boxed{x = y^2}$

$f(y^2, y) = 15y^2 - 4y^2y - 3y^2$

$g(y) = 12y^2 - 4y^3$

$g' = 24y - 12y^2$

$0 = 12y(2 - y)$

$y = 0 \quad y = 2$

$(0,0)$

$(4,2)$

$\boxed{x = 9}$

$g(y) = f(9, y) = 135 - 36y - 3y^2$

$g' = -36 - 6y$

$0 = -36 - 6y$

$36 = -6y$

$-6 = y$

$(9, -6)$ not in Region