Problem 1

A surface is to be oriented in the direction of the positive x−axis. Your friend parameterized the surface and has provided the following cross product. Determine if this cross product could be correct.

(a) cross product: $\langle 1, \ -a, \ b^2 \rangle$

(b) cross product: $\langle 2a, \ 1, \ b+a \rangle$

(a) Since the x component of the vector is a positive one the surface is oriented in the direction of the positive x-axis. Cross product is correct.

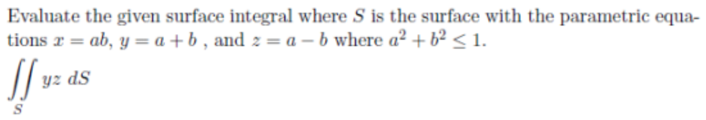
(B) Cross may or may not be correct.

If the values of a are positive this could be correct. Since if $a > 0$ then 2a would be positive.

If the values of a are not allways positive then the cross product would be incorrect.

As the vector is written we know the surface is oriented in the positive y direction.

Problem 2



Evaluate the given surface integral where S is the surface with the parametric equations $x = ab$, $y = a + b$, and $z = a - b$ where $a^2 + b^2 \leq 1$.

$$\iint_S yz\, dS$$

need the cross product.

$r_a = \langle b, 1, 1 \rangle$

$r_b = \langle a, 1, -1 \rangle$

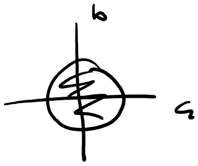
$$r_a \times r_b = \begin{vmatrix} i & j & k \\ b & 1 & 1 \\ a & 1 & -1 \end{vmatrix} = \langle -2, b+a, b-a \rangle$$

$$|r_a \times r_b| = \sqrt{4 + 2b^2 + 2a^2}$$

$$\iint_S yz\, dS = \iint_D (a+b)(a-b)\sqrt{4 + 2b^2 + 2a^2}\; dA$$

Region D

$a^2 + b^2 \leq 1$



$$= \iint_D (a^2 - b^2)\sqrt{4 + 2b^2 + 2a^2}\; dA$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{1} \left(r^2\cos^2\theta - r^2\sin^2\theta\right)\sqrt{4 + 2r^2} \cdot r\, dr\, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{1} \left(\cos^2\theta - \sin^2\theta\right) r^3 \sqrt{4 + 2r^2}\; dr\, d\theta$$

$$= \int_{\theta=0}^{2\pi} \cos^2\theta - \sin^2\theta\; d\theta \cdot \int_{r=0}^{1} r^3\sqrt{4 + 2r^2}\; dr$$

now lets compute each Integral.

$$\int_{\theta=0}^{2\pi} \cos^2\theta - \sin^2\theta \, d\theta = \int_{\theta=0}^{2\pi} \frac{1}{2}(1+\cos 2\theta) - \frac{1}{2}(1-\cos 2\theta)$$

$$= \int_{\theta=0}^{2\pi} \frac{1}{2} + \frac{1}{2}\cos 2\theta - \frac{1}{2} + \frac{1}{2}\cos 2\theta \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \cos 2\theta \, d\theta = \frac{1}{2}\sin(2\theta)\Big|_0^{2\pi} = \frac{1}{2}\sin(4\pi) - \frac{1}{2}\sin(0)$$

$$= 0 - 0 = 0$$

Since This integral is zero, we do not need to compute the other integral.

final Answer is zero

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note this is not a surface area problem so an answer of zero is not a problem.

Problem 3

Let S be the part of the paraboloid $x = y^2 + z^2$ that lies inside the region given by $0 \leq y \leq 1$, $0 \leq z \leq 1$ with orientation in the positive x direction.
Let $\mathbf{F} = \langle e^z,\ ze^y,\ y^2 z\rangle$. Find the flux of $\mathbf{F}$ across S. i.e. compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$

Lets prameterize the surface by

$$r(y,z) = \langle y^2+z^2,\ y,\ z\rangle$$

Then

$$r_y \times r_z = \langle 1, -2y, -2z\rangle$$

Since x term is positive This is the correct orientation.

Since F does not contain the variable x, there is no substitution into the vector field.

$$\iint_S F \cdot dS = \iint_D \langle e^z, ze^y, y^2 z\rangle \cdot \langle 1, -2y, -2z\rangle \, dA$$

$$= \iint_D e^z - 2zye^y - 2y^2z^2 \, dA$$

$$= \int_{y=0}^{1} \int_{z=0}^{1} e^z - 2zye^y - 2y^2z^2 \, dz\,dy$$

$$= \int_{y=0}^{1} \left(e^z - z^2ye^y - \frac{2}{3}y^2z^3\right)\Big|_{z=0}^{1} dy$$

$$= \int_{y=0}^{1} e^1 - ye^y - \frac{2}{3}y^2 - (e^0 - 0 - 0) \, dy$$

$$= \int^{1} e^1 - \underbrace{ye^y} - \frac{2}{3}y^2 - 1 \, dy$$

Integration

$$= \int_{y=0} e - \underbrace{ye}_{} - \frac{2}{3}y - 1 \; dy$$

Integration by parts

D		I
y		e^y
1	$+$	e^y
0	$-$	e^y
	$+\int$	

$$\int ye^y dy = ye^y - e^y$$

$$= ye^1 - \left(ye^y - e^y\right) - \frac{2}{9}y^3 - y \Big|_{y=0}^{1}$$

$$= e^1 - \left(e^1 - e^1\right) - \frac{2}{9} - 1 - \left(0 - (0 - e^0) - 0 - 0\right)$$

$$= e^1 - 0 - \frac{11}{9} - \left(0 + e^0 - 0 - 0\right)$$

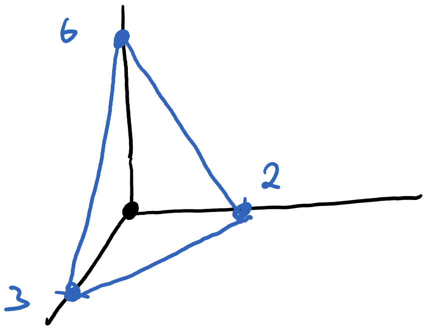
$$= e^1 - \frac{11}{9} - 1 = \underline{e^1 - \frac{20}{9}}$$

Problem 4

Let S be the closed surface of a tetrahedron with vertices $(0,0,0)$, $(3,0,0)$, $(0,2,0)$, and $(0,0,6)$ Let $\mathbf{F} = \langle y, z-y, x\rangle$ and use positive orientation.

Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$

note: using Information from 16.9 will make this problem easier



we need the equation of the plane (blue part of the picture)

remember we can plug the points of the plane into the equation $ax+by+cz=d$

$(3,0,0) \longrightarrow 3a=d$

$(0,2,0) \longrightarrow 2b=d$

$(0,0,6) \longrightarrow 6c=d$

$\longrightarrow$ lets pick $d=6$ and solve for $a, b, + c$

$a=2 \quad b=3 \quad c=1$

equation of the plane $2x+3y+z=6$

This surface has 4 sides so we need to compute 4 surface Integrals.

S_1 plane

S_2 xy plane

S_3 yz plane

S xz plane

positive orientation of the surface of a solid means means normal vectors point away from the solid.

S_3 yz plane

S_4 xz plane.

point away from the solid.

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part 1) S_1 the plane $2x+3y+z=6$

$r(x,y) = \langle x, y, 6-2x-3y \rangle$

$r_x \times r_y = \langle 2, 3, \underline{\underline{1}} \rangle$

orientation is <u>upward</u> so this is the correct orientation..

$F = \langle y, z-y, x \rangle = \langle y, 6-2x-4y, x \rangle$

$$\iint F \cdot dS = \iint_D 2y + 3(6-2x-4y) + x \, dA$$

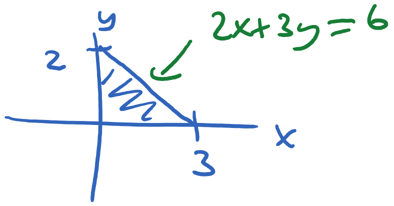
$$= \iint_D 2y + 18 - 6x - 12y + x \, dA$$

$$= \iint_D 18 - 5x - 10y \, dA$$

$$= \int_{x=0}^{3} \int_{y=0}^{\frac{6-2x}{3}} 18-5x-10y \, dy \, dx = 19$$

(done by symbol lab)

Region D is the projection of the plane on the xy plane



Side 2) xy plane. $(z=0)$

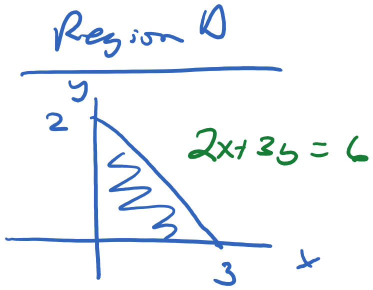
$r(x,y) = \langle x, y, 0 \rangle$

$r_x \times r_y = \langle 0, 0, 1 \rangle$ wrong direction

$r_x \times r_y = \langle 0, 0, 1 \rangle$

use $\langle 0, 0, -1 \rangle$

$F = \langle y, z-y, x \rangle = \langle y, -y, x \rangle$



$$\iint_S F \cdot dS = \iint_D -x \, dA$$

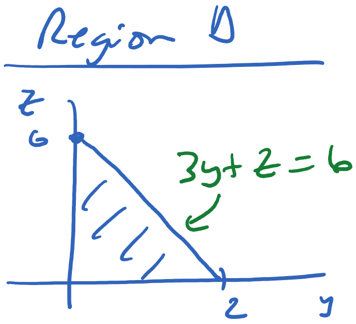
$$= \int_{x=0}^{3} \int_{y=0}^{\frac{6-2x}{3}} -x \, dy \, dx = -3$$ (done by symbol lab)

<u>side 3</u> yz plane $(x=0)$

$r(y,z) = \langle 0, y, z \rangle$

$r_y \times r_z = \langle 1, 0, 0 \rangle$ wrong direction
use $\langle -1, 0, 0 \rangle$

$F = \langle y, z-y, x \rangle = \langle y, z-y, 0 \rangle$



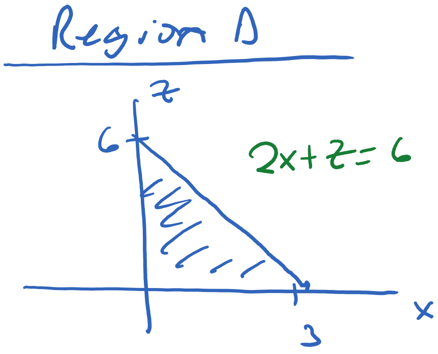
$$\iint_S F \cdot dS = \iint_D -y \, dA$$

$$= \int_{y=0}^{2} \int_{z=0}^{6-3y} -y \, dz \, dy = -4$$ (done by symbol lab)

$z=0$

$$\iint_S F\cdot dS = \iint_D -z\ dA$$

$$= \int_{x=0}^{3}\int_{z=0}^{6-2x} -z\ dz\ dx = -18$$ (done by symbollab)



find answer: $19 + -3 + -4 + -18$

$$= \underline{-6}$$