Problem 1

Let S be the part of the circular cylinder $x^2 + z^2 = 9$ in the first octant such that $1 \le y \le 12$.

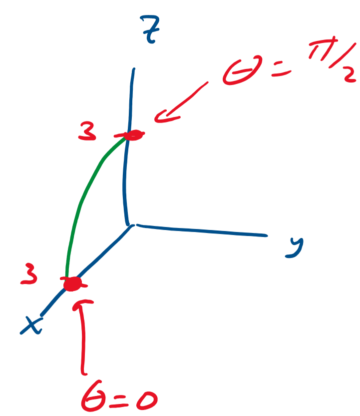
(a) Parameterize S using a polar aspect. Give the intervals for the variables as well as the cross product for the surface.

A polar aspect means we need to think of the circular cylinder as

$$x = r\cos\theta \qquad y = y \qquad z = r\sin\theta$$

Since a surface can only have 2 parameters, one of these letters is a constant. we see $r = 3$.

$$x = 3\cos\theta \qquad y = t \qquad z = 3\sin\theta$$



To figure out the values of θ lets think of the picture if

$\theta = 0$ then $x = 3$ and $z = 0$

if $\theta = \frac{\pi}{2}$ then $x = 0$ & $z = 3$

we want the part of the cylinder in the first octant thus

$$0 \le \theta \le \frac{\pi}{2} \quad \text{and} \quad 0 \le t \le 12$$

$$r(t,\theta) = \langle 3\cos\theta, t, 3\sin\theta \rangle$$

Cross product

$$r_t = \langle 0, 1, 0 \rangle$$

$$r_\theta = \langle -3\sin\theta, 0, 3\cos\theta \rangle$$

$$r_t \times r_\theta = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ -3\sin\theta & 0 & 3\cos\theta \end{vmatrix}$$

$$= (3\cos\theta - 0)\, i - (0-0)\, j + (0 - -3\sin\theta)\, k$$

$$= \langle 3\cos\theta, 0, 3\sin\theta \rangle$$

Problem 1b

(b) Parameterize S using a cartesian method. Give the intervals for the variables as well as the cross product for the surface.

If I project the surface to the xy plane

$$x = x \qquad y = y \qquad \underbrace{z = \sqrt{9 - x^2}}$$

positive root since we want the part above the xy-plane.

Intervals.

$$0 \le x \le 3$$
$$0 \le y \le 12$$

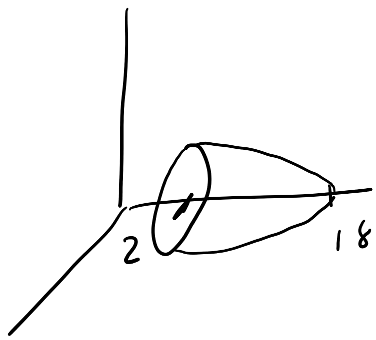
Cross product

$$r_x \times r_y = \langle -f_x , -f_y , 1 \rangle$$

$$= \left\langle \frac{x}{\sqrt{1 - x^2}} , 0 , 1 \right\rangle$$

Problem 2

Let S be the part of the paraboloid $y = 18 - x^2 - z^2$ that is to the right of the plane $y = 2$. Parameterize S using a cartesian method. Give the intervals for the variables as well as the cross product for the surface.

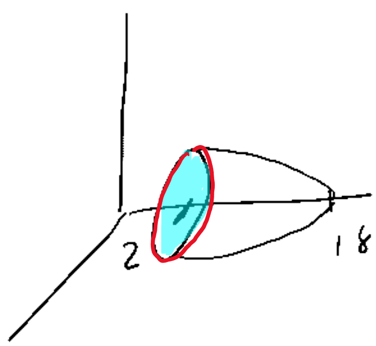


This is the surface.

lets let $x = x$ $z = z$ and $y = 18 - x^2 - z^2$

$$r(x,z) = \langle x, 18 - x^2 - z^2, z \rangle$$

The Interval for the variables is easiest looked at as a region. notice if we project this surface to the xz plane, we are basically projecting the blue disk to the xz plane. The boundary (red edge) of the disk is when $y = 2$ or



$$2 = 18 - x^2 - z^2$$

$$x^2 + z^2 = 16$$

So the interval for the variables is any points on the disk. ie. $x^2 + z^2 \leq 16$

Cross product

$$r_x \times r_z = \langle -f_x, 1, -f_z \rangle$$

$$= \langle -(-2x), 1, -(-2z) \rangle$$
$$= \langle 2x, 1, 2z \rangle$$

Problem 3

Parameterize the surface of the cone $z = \sqrt{x^2 + y^2}$ using spherical.

for this cone we know $\phi = \frac{\pi}{4}$

spherical

we know $\cos\phi = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$\boxed{\text{Answer}}$ $r(\rho\,\theta) = \left\langle \frac{\rho\sqrt{2}}{2}\cos\theta,\ \rho\frac{\sqrt{2}}{2}\sin\phi,\ \frac{\rho\sqrt{2}}{2} \right\rangle$

we know $0 \le \theta \le 2\pi$ and $\rho \geq 0$.

Problem 4

Identify the surface with the given vector equation.

$$\begin{array}{l} \quad\;\;\; x \qquad\;\; y \qquad\quad\;\; z \\ \mathbf{r}(u,v) = \langle u+v, 4-v, 3+4u+6v \rangle \end{array}$$

$x = u + v$

$y = 4 - v$

$z = 3 + 4u + 6v$

$v = 4 - y$

$x = u + 4 - y$

$x + y - 4 = u$

$z = 3 + 4(x + y - 4) + 6(4 - y)$

$z = 3 + 4x + 4y - 16 + 24 - 6y$

$\underbrace{z = 11 + 4z - 2y}$

The surface is a plane.

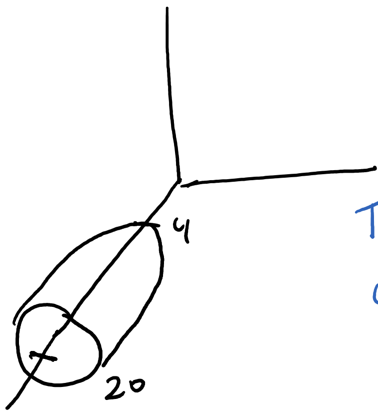
Problem 5

Let S be the part of the paraboloid $x = 4 + y^2 + z^2$ that is inside the cylinder $y^2 + z^2 = 16$. Find the surface area of S.

$x = 4 + y^2 + z^2$

$x = 4 + 16$

$x = 20$



This is the part of the paraboloid inside the cylinder

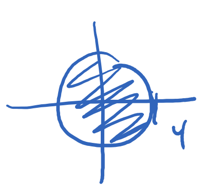
$$r(y,z) = \langle 4 + y^2 + z^2, y, z \rangle$$

we need the cross product

$$\text{cross product} = \langle 1, -2y, -2z \rangle$$

$$SA = \iint_D |\text{cross product}|\, dA = \iint_D \sqrt{1 + 4y^2 + 4z^2}\, dA$$

Region D is on the yz plane



$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{4} r\sqrt{1+4r^2}\, dr\, d\theta$$

$$= \int_0^{2\pi} 1\, d\theta \cdot \underbrace{\int_0^4 r\sqrt{1+4r^2}\, dr}$$

can do u-sub

$= 2\pi \cdot \int$

$$= 2\pi \cdot \int_1^{65} \frac{1}{8}\sqrt{u}\, du$$

$\hookrightarrow$ can do u-sub

$u = 1+4r^2$

$du = 8r\,dr$

$\frac{1}{8}du = r\,dr$

$$= 2\pi \cdot \frac{1}{8}\cdot\frac{2}{3} u^{3/2}\Big|_1^{65} = \underbrace{\frac{\pi}{6}\left[(65)^{3/2} - (1)^{3/2}\right]}$$

Find the area of the surface that is parameterized by $r(a, b) = \langle ab, a + b, a - b\rangle$ with $a^2 + b^2 \le 1$

note: do not do a substitution $a=x$ $b=y$ This is not valid and can and will cause problems in later problems. In this problem we know

$x=ab \quad y=a+b \quad z=a-b$

finding the cartesian surface is not useful.

$$r_a = \langle b, 1, 1\rangle$$

$$r_b = \langle a, 1, -1\rangle$$

$$r_a \times r_b = \begin{vmatrix} i & j & k \\ b & 1 & 1 \\ a & 1 & -1 \end{vmatrix} = \langle -2, b+a, b-a\rangle$$

$$|r_a \times r_b| = \sqrt{(-2)^2 + (b+a)^2 + (b-a)^2}$$

$$= \sqrt{4 + b^2 + 2ab + a^2 + b^2 - 2ab + a^2}$$

$$= \sqrt{4 + 2b^2 + 2a^2}$$

$$SA = \iint_D |r_a \times r_b|\, dA = \iint_D \sqrt{4 + 2b^2 + 2a^2}\, dA$$

Region D
$a^2 + b^2 \le 1$

$$= \int_0^{2\pi}\int_0^1 r\sqrt{4+2r^2}\, dr\, d\theta = \int_0^{2\pi} d\theta \cdot \int_0^1 r\sqrt{4+2r^2}\, dr$$

$$= \int_{\theta=0} \int_{r=0} r\sqrt{4+2r^2}\, dr\, d\theta = \int_{\theta=0} d\theta \cdot \underbrace{\int_{r=0} r\sqrt{4+2r^2}\, dr}$$

Let

$u = 4+2r^2$

$du = 4r\,dr$

$\frac{1}{4}\, du = r\,dr$

$$= 2\pi \cdot \int_{u=4}^{6} \frac{1}{4}\sqrt{u}\, du = \frac{2\pi}{4} \cdot \frac{2}{3} u^{3/2} \Big|_4^6$$

$$= \frac{\pi}{3}\left[6^{3/2} - 4^{3/2}\right] = \frac{\pi}{3}\left[6^{3/2} - 8\right]$$