Problem 1

if curl F = $\langle 0,0,0\rangle$ then conservative

Determine if the vector field is consevative.

(a) $\mathbf{F} = \langle 4yz,\ 1 - 6yz^3,\ 4x^2 - 9y^2z^2\rangle$

$$\text{Curl } F = \langle -18yz^2 + 18yz^2,\ 4y - 8x,\ 0 - 4z\rangle$$
$$= \langle 0,\ 4y - 8x,\ -4z\rangle$$

F is <u>not</u> conservative

(b) $\mathbf{F} = \langle 8xz,\ 1 - 6yz^3,\ 4x^2 - 9y^2z^2\rangle$

$$\text{curl } F = \langle -18yz^2 + 18yz^2,\ 8x - 8x,\ 0 - 0\rangle$$
$$= \langle 0,\ 0,\ 0\rangle$$

F is conservative

Problem 2

If $\mathbf{F} = \langle 8xz,\ 1 - 6yz^3,\ 4x^2 - 9y^2z^2 \rangle$, compute $\nabla \cdot F$

$\boxed{\text{note } \nabla \times F = \operatorname{curl} F}$

$$\nabla \cdot F = \operatorname{div} F$$

$$= P_x + Q_y + R_z$$

$$= 8z - 6z^3 - 18y^2z$$

Problem 3

For the vector field $\mathbf{F} = \langle y^2z^3,\ 2xyz^3,\ 3xy^2z^2 \rangle$. Determine if the vector field is conservative. If it is conservative, then compute the following line integral.

$R_y - Q_z,\ P_z - R_x,\ Q_x - P_y$

Compute $\int_{(1,1,1)}^{(2,2,1)} \mathbf{F} \cdot dr$

$$\text{Curl}\,F = \langle 6xyz^2 - 6xyz^2,\ 3y^2z^2 - 3y^2z^2,\ 2yz^3 - 2yz^3 \rangle$$

$$= \langle 0, 0, 0 \rangle$$ Thus F is conservative.

$f_x = y^2z^3 \longrightarrow f(x,y,z) = \underline{xy^2z^3} + c(y,z)$

$f_y = 2xyz^3 \longrightarrow f(x,y,z) = \underline{xy^2z^3} + c(x,z)$

$f_z = 3xy^2z^2 \longrightarrow f(x,y,z) = \underline{xy^2z^3} + c(x,y)$

Thus $f(x,y,z) = xy^2z^3$

$$\int_{(1,1,1)}^{(2,2,1)} F \cdot dr = f(2,2,1) - f(1,1,1)$$

$$= 2 \cdot 4 \cdot 1 - 1 \cdot 1 \cdot 1 = 8 - 1 = 7$$