Problem 1

Given $F = \langle 2xy^3, 3x^2y^2 \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve given by $\mathbf{r}(t) = \langle t^3 + 2t^2 - t, 3t^4 - t^2 \rangle$, $0 \le t \le 1$.

F is a continuous vector field on the xy plane. (Thus this Region is an open connected Region.)

$P = 2xy^3 \qquad Q = 3x^2y^2$

$\underline{P_y = 6xy^2} \qquad \underline{Q_x = 6xy^2}$

equal Thus F is conservative.

$f_x = 2xy^3 \longrightarrow f(x,y) = x^2y^3 + C(y)$

$f_y = 3x^2y^2 \longrightarrow f(x,y) = x^2y^3 + C(x)$

Thus $f(x,y) = x^2y^3$

$r(t) = \langle t^3 + 2t^2 - t, 3t^4 - t^2 \rangle \qquad 0 \le t \le 1$

$r(1) = \langle 2, 2 \rangle$

$r(0) = \langle 0, 0 \rangle$

$$\int F \cdot dr = \int^{(2,2)} \nabla f \cdot r'(t)\, dr = f(2,2) - f(0,0)$$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_{(0,0)} \nabla f \cdot \mathbf{r}'(t)\, dr = f(2,2) - f(0,0)$$

$$= (2)^2(2)^3 - 0 = 32$$

Problem 2

$P \quad Q$

Let $\mathbf{F} = \langle 2x + y^3,\ 3xy^2 + 4\rangle$. Evaluate $\displaystyle\int_{(0,1)}^{(2,3)} \mathbf{F} \cdot d\mathbf{r}$.

F is a continuous vector field with domain being the xy plane.

The xy plane is an open simply connected Region.

$P_y = 3y^2 \qquad Q_x = 3y^2 \qquad$ so since $P_y = Q_x$, F is conservative.

$f_x = 2x + y^3 \longrightarrow f(x,y) = x^2 + xy^3 + c(y)$

$f_y = 3xy^2 + 4 \longrightarrow f(x,y) = xy^3 + 4y + c(x)$

$f(x,y) = x^2 + xy^3 + 4y$

$$\int_{(0,1)}^{(2,3)} F \cdot dr = f(2,3) - f(0,1)$$

$$= 4 + 2(27) + 12 - (0 + 0 + 4)$$

$$= 4 + 54 + 12 - 4 = 66$$

Problem 3

Given $F = \langle y^2\cos(x),\ 2y\sin(x) + e^{2z},\ 2ye^{2z}\rangle$ is a conservative vector field. Find the work done by F when moving a particle on any path C from from the point $(0, 1, \frac{1}{2})$ to the point $(\frac{\pi}{2}, 3, 2)$.

After learning section 16.5, see if you can show that this vector field is conservative.

$$f_x = y^2\cos(x) \rightarrow f(x,y,z) = y^2\sin(x) + c(y,z)$$

$$f_y = 2y\sin(x) + e^{2z} \rightarrow f(x,y,z) = y^2\sin(x) + ye^{2z} + c(x,z)$$

$$f_z = 2ye^{2z} \longrightarrow f(x,y,z) = ye^{2z} + c(x,y)$$

$$f(x,y,z) = y^2\sin(x) + ye^{2z}$$

$$\int_C F\cdot dr = f\left(\frac{\pi}{2}, 3, 2\right) - f\left(0, 1, \frac{1}{2}\right)$$

$$= 9\sin\left(\frac{\pi}{2}\right) + 3e^4 - \left[1\sin(0) + 1e^1\right]$$

$$= 9 + 3e^4 - e^1$$