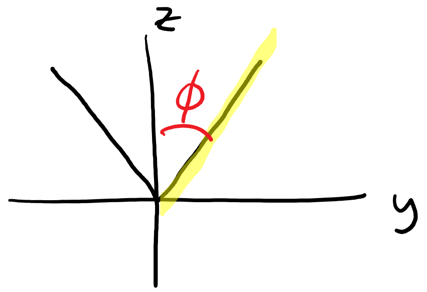
Problem 1

Convert these equations to sperical coordinates.

(a) $z = \sqrt{5x^2 + 5y^2}$

notice that this is the top of a cone.

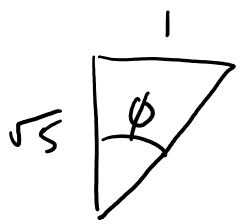
Let $x=0$ below is the cross section of the cone when $x=0$.



if $x=0$ then $z = \sqrt{0+5y^2}$

$z = \sqrt{5}\,y$ (This is the equation of the yellow line.)

pick $y=1$ Then $z=\sqrt{5}$

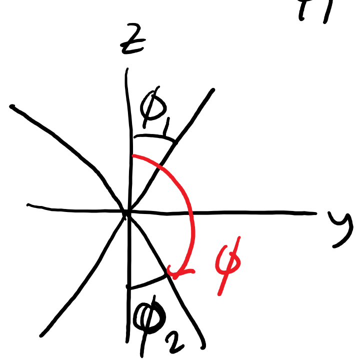


Thus $\tan\phi = \frac{1}{\sqrt{5}}$

$\phi = \arctan\left(\frac{1}{\sqrt{5}}\right)$

(b) $z = -\sqrt{7x^2 + 7y^2}$ This is the bottom part of a cone.

Cross section of the full cone lets us see that ϕ_1 and ϕ_2 are equal. and that

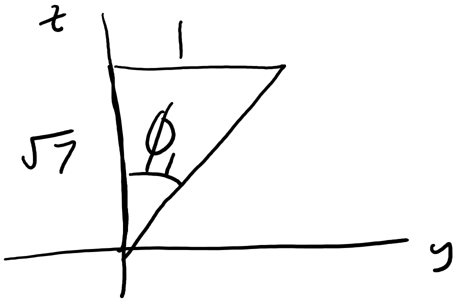


$\phi = \pi - \phi_2 = \pi - \phi_1$

So lets ignore the negative sign for now and solve for ϕ_1

Let $x=0$ $\quad z = \sqrt{0+7y^2} = \sqrt{7}\, y^2$

pick $y=1$ so $z=\sqrt{7}$



$$\tan\phi_1 = \frac{1}{\sqrt{7}}$$

$$\phi_1 = \arctan\left(\frac{1}{\sqrt{7}}\right)$$

Thus $\phi = \pi - \arctan\left(\frac{1}{\sqrt{7}}\right)$

Problem 2

Convert the integral to sperical.

$$\int_0^{1.5}\int_{x\sqrt{3}}^{\sqrt{9-x^2}}\int_{-\sqrt{36-x^2-y^2}}^{-\sqrt{3x^2+3y^2}} z\sqrt{x^2+y^2+z^2}\ dzdydx$$

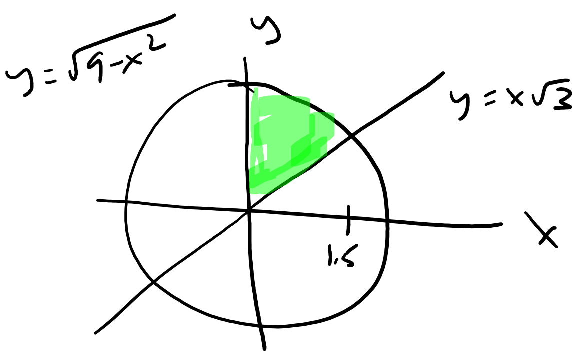
Top $z = -\sqrt{3x^2+3y^2}$ Bottom of a cone

Bottom $z = -\sqrt{36-x^2-y^2}$ Bottom of a sphere

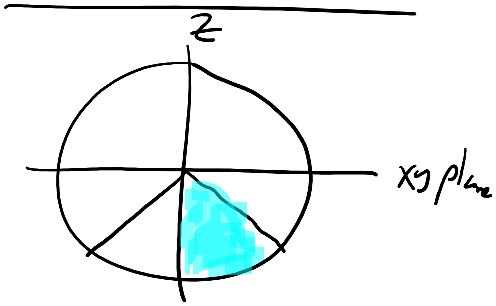
Region D

$x\sqrt{3} \le y \le \sqrt{9-x^2}$

$0 \le x \le 1.5$



overall shape



verification that $x=1.5$ Is The intersection value.

$y=x\sqrt{3}$ $\quad x^2+y^2=9$

$x^2+(x\sqrt{3})^2=9$

$x^2+3x^2=9$

$4x^2=9$

$x^2=\frac{9}{4}$

$x=\pm\frac{3}{2}=\pm 1.5$

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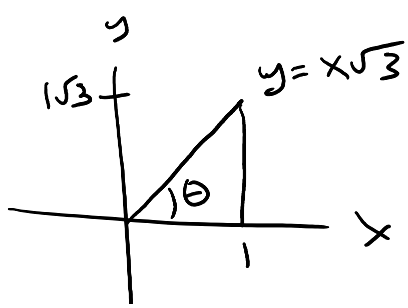
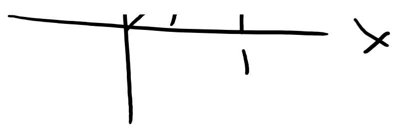
now find Theta values



$\tan\theta = \frac{1\sqrt{3}}{1} = \sqrt{3}$

$\phi = \frac{\pi}{3}$



Thus the interval for θ is $\frac{\pi}{3} \le \theta \le \frac{\pi}{2}$

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now find the value for ϕ for the cone.

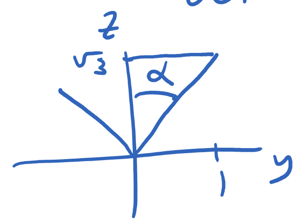
$z = -\sqrt{3x^2+3y^2}$ $\longrightarrow$ Lets look at $z=\sqrt{3x^2+3y^2}$

let $x=0$

then

$z=\sqrt{3}\,y$

pick $y=1$ then $z=\sqrt{3}$



$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\phi = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Thus $\frac{5\pi}{6} \le \phi \le \pi$

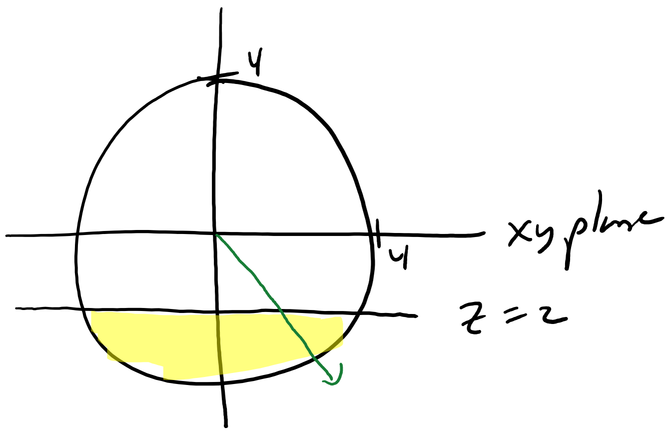
and $0 \le \rho \le 6$

Answer

$$\int_{\theta=\frac{\pi}{3}}^{\pi/2} \int_{\phi=\frac{5\pi}{6}}^{\pi} \int_{\rho=0}^{6} \rho\cos\phi \cdot \rho \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

Problem 3

Set up the integral, in spherical, to find the volume of the region that is inside a sphere(centered at the origin) of radius 4 and below the plane $z = -2$.



$0 \le \theta \le 2\pi$

$-2\sec\phi \le \rho \le 4$

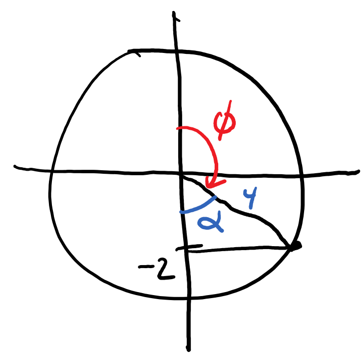
$\frac{2\pi}{3} \le \phi \le \pi$

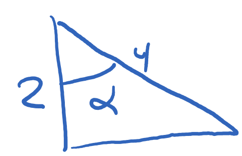
$z = -2$

$\rho\cos\phi = -2$

$\rho = -2\sec\phi$

now find ϕ





$\cos\alpha = \frac{2}{4} = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$

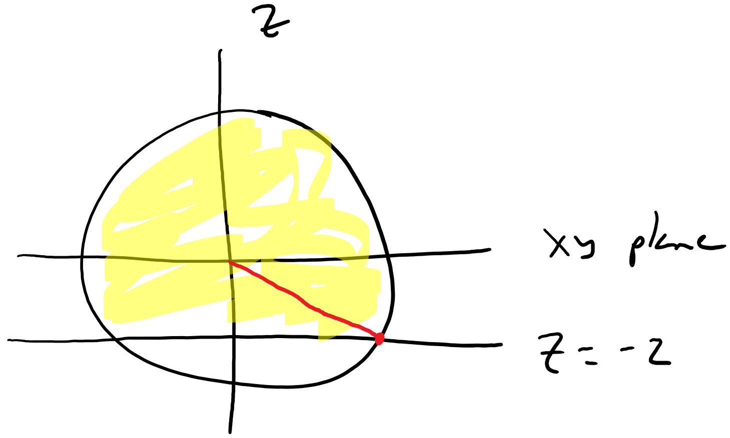
$\phi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Answer:

$$V = \iiint_E 1 \, dV = \int_{\theta=0}^{2\pi} \int_{\phi=\frac{2\pi}{3}}^{\pi} \int_{\rho=-2\sec\phi}^{4} 1 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

Problem 4

Set up the integral, in spherical, to find the volume of the region that is inside a sphere(centered at the origin) of radius 4 and above the plane $z = -2$. **Note: be very careful with this problem.**



from the previous problem we know the ϕ value for the red line is

$$\phi = \frac{2\pi}{3}$$

The plane $z = -2$ is $\rho = -2\sec\phi$

notice the value of ρ depends on the interval of ϕ.

for $0 \le \phi \le \frac{2\pi}{3}$, $0 \le \rho \le 6$ and $\frac{2\pi}{3} \le \phi \le \pi$, $0 \le \rho \le -2\sec\phi$

$$V = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\frac{2\pi}{3}} \int_{\rho=0}^{6} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta + \int_{\theta=0}^{2\pi} \int_{\phi=\frac{2\pi}{3}}^{\pi} \int_{\rho=0}^{-2\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$