Problem 1

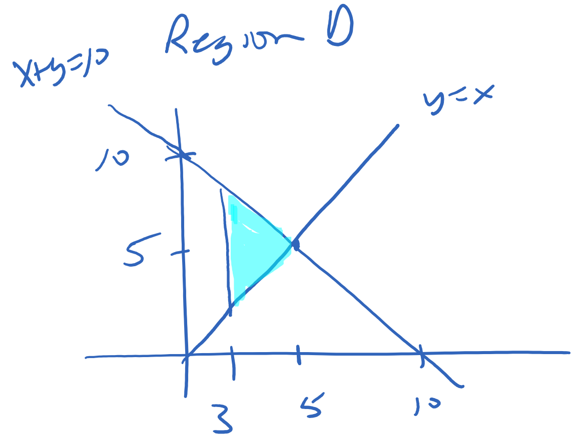
Find the surface area of the part of the plane $4x + 2y - z + 5 = 0$ that lies above the region in the xy-plane bounded by $x + y = 10$, $y = x$ and $x = 3$.

$$SA = \iint_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

$$f(x,y) = z = 4x + 2y + 5$$

$$f_x = 4$$

$$f_y = 2$$



$$SA = \iint_D \sqrt{4^2 + 2^2 + 1}\; dA$$

$$= \iint_D \sqrt{21}\, dA$$

from the graph of Region D we want this to be a dy dx Integral

$$3 \le x \le 5$$

$$x \le y \le 10 - x$$

$$= \int_{x=3}^{5} \int_{y=x}^{10-x} \sqrt{21}\, dy\, dx$$

$$= \int_{x=3}^{5} y\sqrt{21} \Big|_{x}^{10-x} dx = \int_{x=3}^{5} (10-x)\sqrt{21} - x\sqrt{21} \quad dx$$

$$= \int_{x=3}^{5} 10\sqrt{21} - x\sqrt{21} - x\sqrt{21}\, dx = \int_{x=3}^{5} 10\sqrt{21} - 2x\sqrt{21}\; dA$$

$$= \left(10x\sqrt{21} - x^2\sqrt{21}\right) \Big|_{x=3}^{5}$$

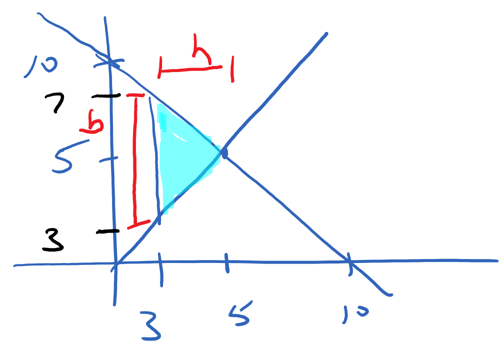
$$= 50\sqrt{21} - 25\sqrt{21} - \left(30\sqrt{21} - 9\sqrt{21}\right)$$

$$= 25\sqrt{21} - 21\sqrt{21}$$

$$= 4\sqrt{21}$$

Notice! Region D is a Triangle.

Region D



Notice:

$$SA = \iint_D \sqrt{21}\, dA$$

$$= \sqrt{21} \underbrace{\iint_D 1\, dA}$$

This means the Area of Region D.

$$= \sqrt{21} \cdot \frac{1}{2} bh$$

base $= 7 - 3 = 4$

$h = \quad 5 - 3 = 2$

$$= \sqrt{21} \cdot \frac{1}{2}(4)(2)$$

$$= 4\sqrt{21}$$

Same answer but no Integration is needed.

Problem 2

Set up the double integral , in polar, that would give the surface area of the part of the ellipsoid $4x^2 + 4y^2 + z^2 = 16$ that is above the plane $z = 2$.

$$f(x,y) = z = +\sqrt{16 - 4x^2 - 4y^2}$$

Positive square root since the ellipsoid is about the origin and $z = 2$ is above the xy-plane.

$$f_x = \frac{-8x}{2\sqrt{16-4x^2-4y^2}} = \frac{-4x}{\sqrt{16-4x^2-4y^2}}$$

$$f_y = \frac{-4y}{\sqrt{16-4x^2-4y^2}}$$

$$SA = \iint_D \sqrt{\left(\frac{-4x}{\sqrt{16-4x^2-4y^2}}\right)^2 + \left(\frac{-4y}{\sqrt{16-4x^2-4y^2}}\right)^2 + 1}\; dA$$

$$= \iint_D \sqrt{\frac{16x^2}{16-4x^2-4y^2} + \frac{16y^2}{16-4x^2-4y^2} + 1}\; dA$$

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now find the Intersection of the ellipsoid + the plane $z = 2$.

$$4x^2 + 4y^2 + (2)^2 = 16$$

or $\quad 4x^2 + 4y^2 + 4 = 16$

$$4x^2 + 4y^2 = 12$$

$$x^2 + y^2 = 3$$

The part of the ellipsoid that we want the

Surface Area for is above the plane $z=2$

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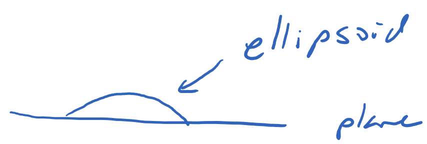


If we project (ie squish) this part to the xy-plane then the region covered on the xy-plane will be inside

$x^2+y^2=3$ This is Region D

$0 \le \theta \le 2\pi$
$0 \le r \le \sqrt{3}$

$$\iint_D \sqrt{\frac{16x^2+16y^2}{16-4x^2-4y^2}+1}\, dA$$ from above

$16x^2+16y^2 = 16r^2$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{3}} \sqrt{\frac{16r^2}{16-4r^2}+1} \cdot r\, dr\, d\theta$$

$16-4x^2-4y^2$
$= 16-4r^2$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{3}} r\sqrt{\frac{16r^2}{16-4r^2}+1}\; dr\, d\theta$$

Since this is just setup we can stop at this point.

Problem 3

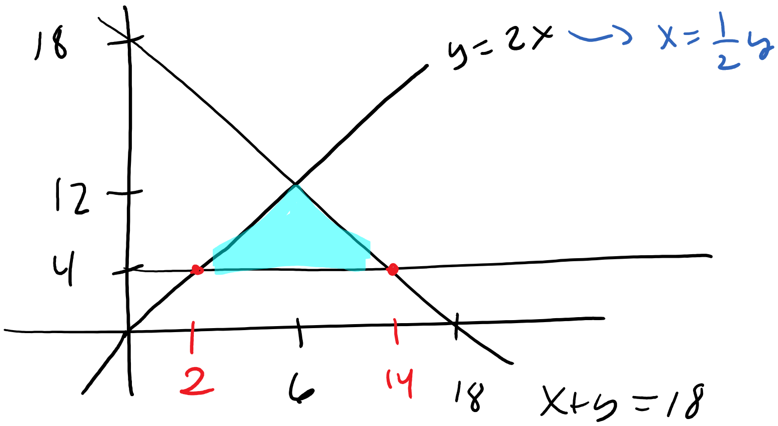
Set up the integral to find the surface area of the portion of the function $z = x^2 + y^5$ above the region in the xy-plane that is bounded by $x + y = 18$, $y = 2x$, and $y = 4$.

$y = 2x \qquad x + y = 18$

$x + 2x = 18$

$3x = 18$

$x = 6$



$x + y = 18$

$\downarrow$

$x = 18 - y$

Based on the Region

a dxdy Integral would be best.

$4 \le y \le 12$

$\frac{1}{2}y \le x \le 18 - y$

$f(x,y) = z = x^2 + y^5$

$f_x = 2x$

$f_y = 5y^4$

$$SA = \iint_D \sqrt{(2x)^2 + (5y^4)^2 + 1}\; dA$$

$$= \int_{y=4}^{12} \int_{x=\frac{1}{2}y}^{18-y} \sqrt{4x^2 + 25x^8 + 1}\; dx\, dy$$