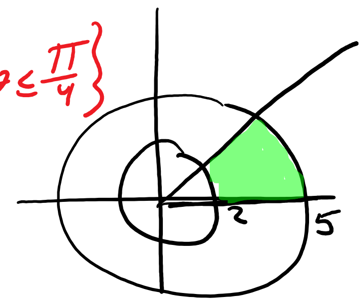
Problem 1

Set up and compute the polar integral to evaluate the following double integral where the region R is in the first quadrant and is bounded by $x^2 + y^2 = 25$, $x^2 + y^2 = 4$, $y = x$ and $y = 0$.

$$\iint_R x \, dA$$

The green Region is R.

graph of R.



$R = \{ (r,\theta) \mid 2 \le r \le 5 \text{ and } 0 \le \theta \le \frac{\pi}{4} \}$

$$\iint_R x \, dA = \int_{\theta=0}^{\pi/4} \int_{r=2}^{5} r\cos\theta \cdot r \, dr \, d\theta$$

$$= \int_{\theta=0}^{\pi/4} \int_{r=2}^{5} r^2 \cos\theta \, dr \, d\theta$$

$$= \int_{\theta=0}^{\pi/4} \cos\theta \, d\theta \cdot \int_{r=2}^{5} r^2 \, dr$$

by Fubini.

$$= \sin\theta \Big|_0^{\pi/4} \cdot \frac{1}{3} r^3 \Big|_2^5$$

$$= \left[\sin\left(\frac{\pi}{4}\right) - \sin(0)\right] \cdot \left[\frac{1}{3}5^3 - \frac{1}{3}2^3\right]$$

$$= \left(\frac{\sqrt{2}}{2} - 0\right)\left(\frac{125}{3} - \frac{8}{3}\right) = \frac{\sqrt{2}}{2} \cdot \frac{117}{3}$$

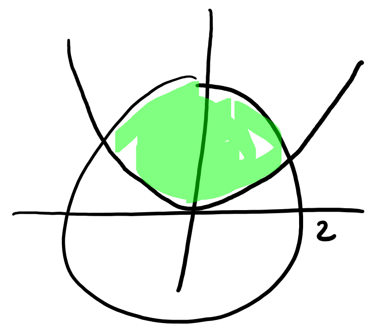
$$= \frac{117\sqrt{2}}{6}$$

Problem 2

Should this integral be computed converting it to a polar integral?

$$\iint_D y\, dA \text{ where } D = \{(x,y) | y \geq x^2,\ \ x^2 + y^2 \leq 4\}$$

Region D

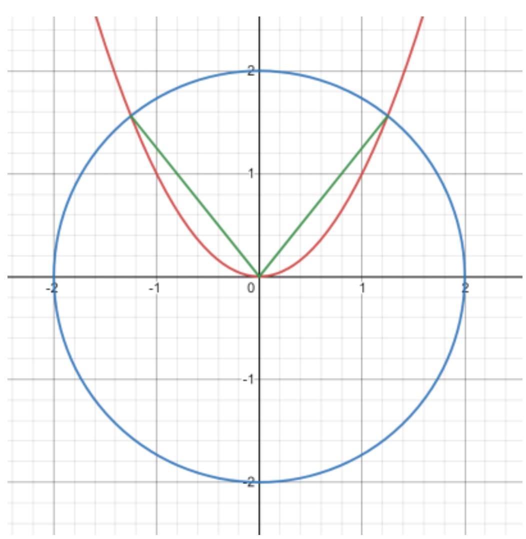


The short answer is

NO!!!

$y = x^2$ is a parabola. It does not convert easily to polar.

If (and I do not sugest it) you do do polar Then you need to Break the Region D into 3 parts.



The green line represent The line from The origin to where the circle and parabola Intersect

$y = 1.25x$ and

$y = -1.25x$

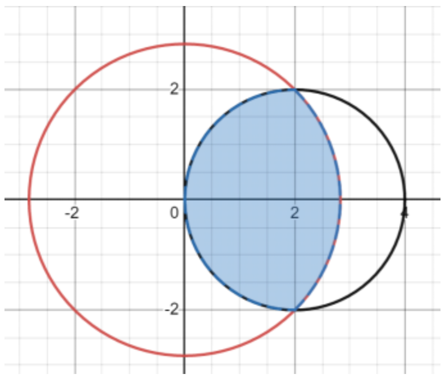
All of this would need to be converted to polar.

parabolas are not function that should be used with polar.

Problem 3

Set up the integral that will compute the following integral over the region on the xy−plane that is inside both of the circles: $x^2 + y^2 = 4x$ and $x^2 + y^2 = 8$.

$$\iint_D 5x + y \, dA$$

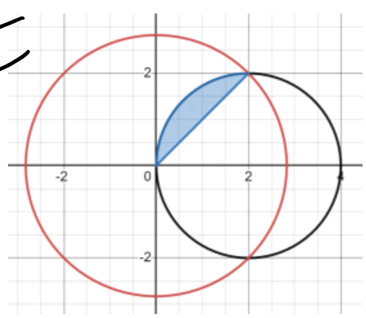


here is the region that we want to integrate over.

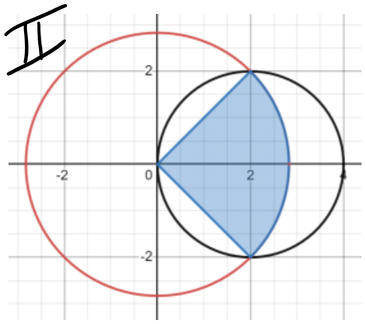
From this picture we see that there is actually 3 region that need to be looked at if we do polar.

notice any arrow drawn outward from the origin would be in one of these 3 regions.

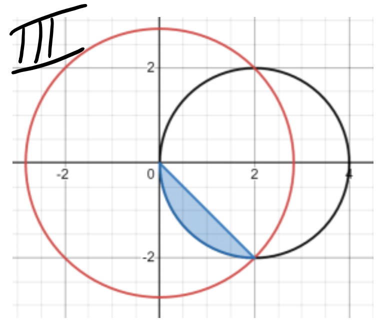
I



II



III



Step 1) Find the angle for intersection.

$x^2+y^2=4x$

$r^2 = 4r\cos\theta$

$r = 4\cos\theta$

$x^2+y^2=8$

$r^2=8$

$r=\sqrt{8}$

$\sqrt{8} = 4\cos\theta$

$\cos\theta = \frac{\sqrt{8}}{4} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$

$$\theta = \frac{\pi}{4} \quad \left(\text{and } -\frac{\pi}{4}\right)$$

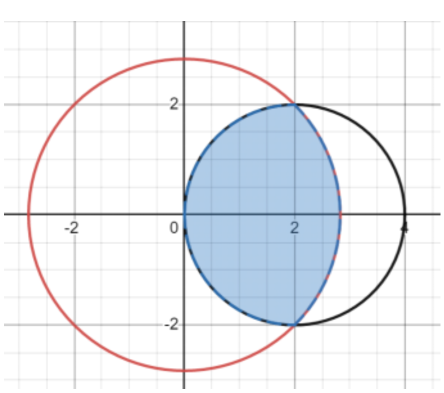
region I	Region II	Region III
$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$	$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$	$-\frac{\pi}{2} \le \theta \le -\frac{\pi}{4}$
$0 \le r \le 4\cos\theta$	$0 \le r \le \sqrt{8}$	$0 \le r \le 4\cos\theta$

Answer

$$\int_{\theta=\pi/4}^{\pi/2} \int_{r=0}^{4\cos\theta} (5r\cos\theta + r\sin\theta)\, r\, dr\, d\theta + \int_{\theta=-\frac{\pi}{4}}^{\pi/4} \int_{r=0}^{\sqrt{8}} (5r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

$$+ \int_{\theta=-\frac{\pi}{2}}^{-\pi/4} \int_{r=0}^{4\cos\theta} (5r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

Cartesian method.



$x^2+y^2=4x$

$y^2 = 4x - x^2$

$y = \pm\sqrt{4x-x^2}$

$x^2+y^2=8$

$y^2 = 8-x^2$

$y = \pm\sqrt{8-x^2}$

Intersect at $\quad 8 = 4x$

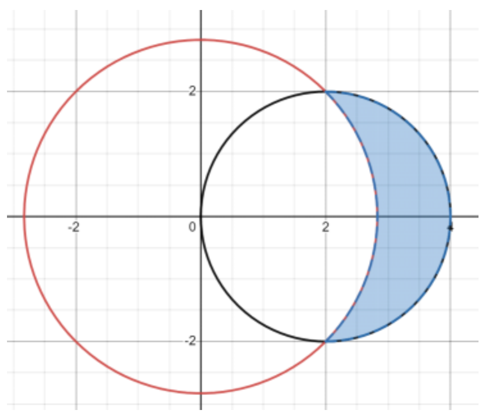
$x = 2$

$$\int_{x=0}^{2} \int_{y=-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}} 5x+y \, dy \, dx + \int_{x=2}^{\sqrt{8}} \int_{y=-\sqrt{8-x^2}}^{\sqrt{8-x^2}} 5x+y \, dy \, dx$$

Problem 4

Set up the integral that will compute the following integral over the region on the xy–plane that is inside the circle $x^2 + y^2 = 4x$ and outside the circle $x^2 + y^2 = 8$.

$$\iint_D 5x + y \, dA$$



from the last problem we Know the graphs Intersect at $\theta = \pi/4$ and $\theta = \frac{-\pi}{4}$

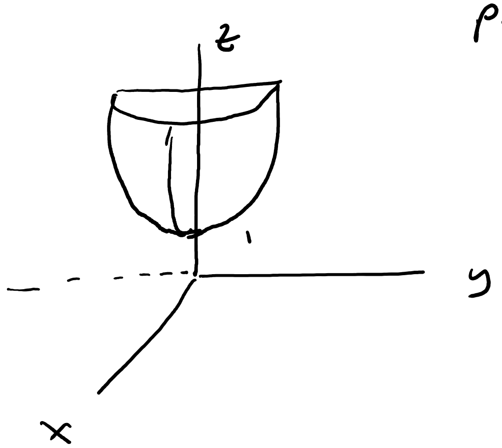
$$\int_{\theta=-\frac{\pi}{4}}^{\pi/4} \int_{r=\sqrt{8}}^{4\cos\theta} (5r\cos\theta + r\sin\theta) r \, dr \, d\theta$$

Problem 5

Setup and compute the double integral(in polar) that would give the volume of the solid bounded by the paraboloid $\underbrace{z = 1 + 2x^2 + 2y^2}$ and the plane $z = 7$ with the condition that $x \geq 0$.

horizontal plane.

paraboloid That starts at $z=1$ and goes up.



note we have $x \geq 0$ so only $\frac{1}{2}$ of the solid.

$\iint_D 7\, dA$ is the volume under the horizontal plane $z=7$ and above the xy-plane on Region D.

$\iint_D 1+2x^2+2y^2\, dA$ is the volume under the paraboloid & the xy-plane on Region D.

The Volume between the surface is what I want so

$$V = \iint_D 7\, dA - \iint_D 1+2x^2+2y^2\, dA$$

top − Bottom

$$V = \iint_D 7 - (1+2x^2+2y^2)\, dA$$

$$= \iint_D \underline{6 - 2x^2 - 2y^2}\, dA$$

$\to 6 - 2(x^2+y^2)$

What is Region D?

Region D is below Both of these curves.

So Intersect

$$7 = 1 + 2x^2 + 2y^2$$

$$6 = 2x^2 + 2y^2$$

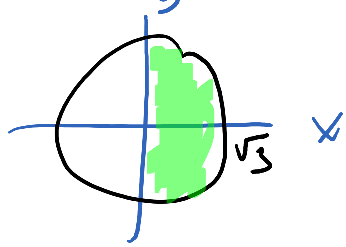
$$3 = x^2 + y^2$$

ie The part of the xy plane covered by $x^2+y^2=3$

Graph of D

$$\iint_D \underbrace{6 - 2x^2 - 2y^2}_{6-2(x^2+y^2)}$$

graph of D



note $x \geq 0$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$0 \leq r \leq \sqrt{3}$

$$V = \int_{-\pi/2}^{\pi/2} \int_0^{\sqrt{3}} (6 - 2r^2)\, r\, dr\, d\theta$$

$$V = \int_{-\pi/2}^{\pi/2} 1\, d\theta \cdot \int_0^{\sqrt{3}} 6r - 2r^3\, dr$$

$$= \theta \Big|_{-\pi/2}^{\pi/2} \cdot \left[3r^2 - \frac{2r^4}{4} \right]_0^{\sqrt{3}}$$

$$= \left(\frac{\pi}{2} - -\frac{\pi}{2}\right) \cdot \left[3(\sqrt{3})^2 - \frac{1}{2}(\sqrt{3})^4 - (0) \right]$$

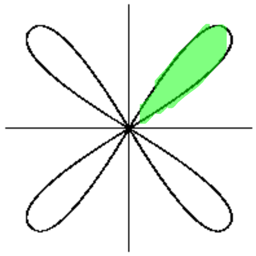
$$= \pi \cdot \left(9 - \frac{1}{2} \cdot 9 \right)$$

$$= 4.5\pi$$

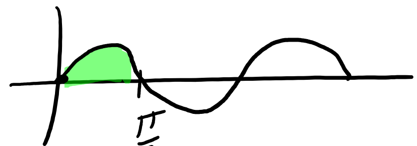
Problem 6

Lets use the green leaf.

Set up the integral to find the volume under the function $f(x, y) = 3x^2 + y$ over the interior of one leaf of $r = \sin(2\theta)$. **Picture is not drawn to scale**



The graph of $y = \sin 2\theta$ is

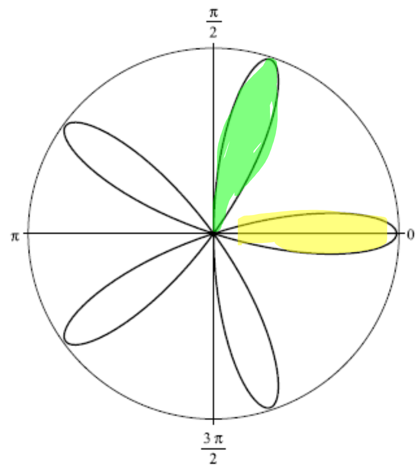


$0 \leq \theta \leq \frac{\pi}{2}$ $\qquad$ $0 \leq r \leq \sin 2\theta$

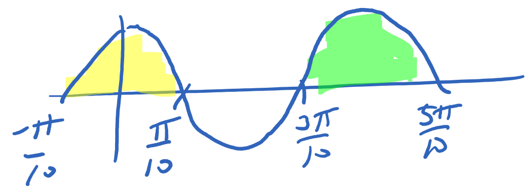
$$\iint_D 3x^2 + y \, dA = \int_{\theta=0}^{\pi/2} \int_{r=0}^{\sin 2\theta} \left(3r^2\cos^2\theta + r\sin\theta\right) r \, dr \, d\theta$$

Problem 7

Set up the integral to find the volume under the function $f(x, y) = 3y^2$ over the interior of one leaf of $r = \cos(5\theta)$. **Ignore the outer circle in the graph. The computer was being helpful.**



graph of $y = \cos 5\theta$



yellow

$$V = \iint_D 3y^2\, dA = \int_{-\pi/10}^{\pi/10} \int_0^{\cos 5\theta} 3r^2 \sin^2\theta \cdot r\, dr\, d\theta$$

green

$$V = \iint_D 3y^2\, dA = \int_{3\pi/10}^{5\pi/10} \int_0^{\cos 5\theta} 3r^2 \sin^2\theta \cdot r\, dr\, d\theta$$