Problem 1

1. Evaluate the integral where $D = \{(x, y) \in \Re^2 \mid x^2 + y^2 \leq 16\}$.

$$\iint_D \sqrt{16 - x^2 - y^2}\, dA$$

$\sqrt{16-x^2-y^2}$ is the top $\frac{1}{2}$ of a sphere. (hemisphere)

since Region D is the base of the hemisphere

This Integral represents $\frac{1}{2}$ Area of a sphere.

$$\iint_D \sqrt{16-x^2-y^2}\, dA = \frac{1}{2} \cdot \frac{4}{3} \pi (4)^3 = \frac{2}{3} \pi \cdot 64$$

$$= \frac{128\pi}{3}$$

Problem 2

2. Set up the double integral that will compute $\iint_D f(x,y)dA$ where D is the region bounded by the curves $y = x^2$ and $\underbrace{x = y - 6}$. $\longrightarrow$ $y = x+6$

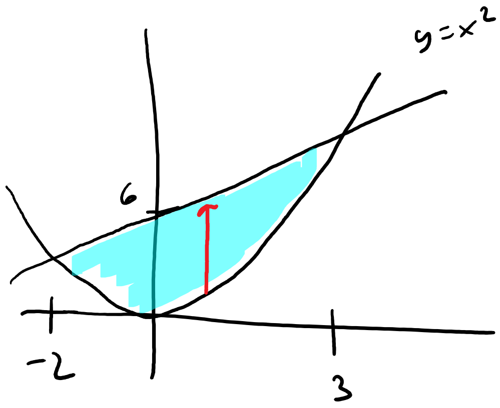
find the intersection points

$x^2 = x+6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$, $x = -2$



notice that this is a type 1 Region ($\iint dy\,dx$)

$-2 \le x \le 3$

$x^2 \le y \le x+6$

$$\iint_D f(x,y)\,dA = \underbrace{\int_{x=-2}^{3} \int_{y=x^2}^{x+6} f(x,y)\,dy\,dx}$$

Problem 3

3. Evaluate the double integral of $f(x, y) = x$ over the region $D = \{(x, y)|\ 0 \le x \le \pi, 0 \le y \le \sin(x)\}$.

Region D does not need to be graphed since we are told the interval for x + y

$$\int_{x=0}^{\pi} \int_{y=0}^{\sin(x)} x \, dy \, dx = \int_{x=0}^{\pi} xy \Big|_{y=0}^{\sin(x)} dx$$

$$= \int_{x=0}^{\pi} x \sin(x) \, dx = x\cos(x) - (-\sin(x)) \Big|_{x=0}^{\pi}$$

$$= -x\cos(x) + \sin(x) \Big|_{0}^{\pi}$$

$$= -\pi \cos(\pi) + \sin(\pi) - (0 + \sin(0))$$

$$= \pi + 0 - 0$$

$$= \boxed{\pi}$$

use the tabular method for Integration by parts.

D	I
x	$\sin(x)$
1	$-\cos(x)$
0	$-\sin(x)$

$+$ / $-$ / $+\int$

Problem 4

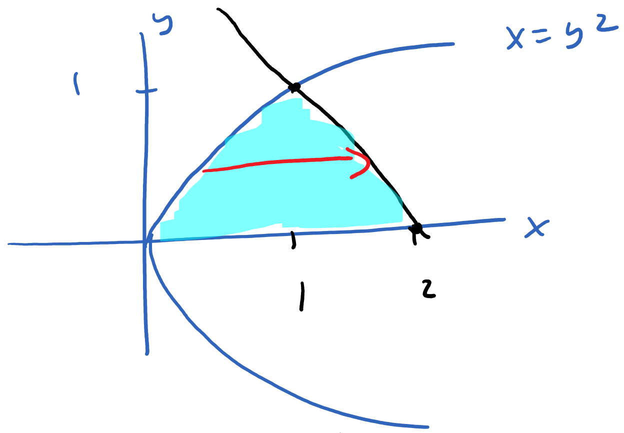
4. Example Change the order of integration.

$$\int_0^1 \int_{y^2}^{2-y} f(x,y)\,dx\,dy$$

$y^2 \le x \le 2-y$

$0 \le y \le 1$

Step 1: write the interval and draw Region D.



line

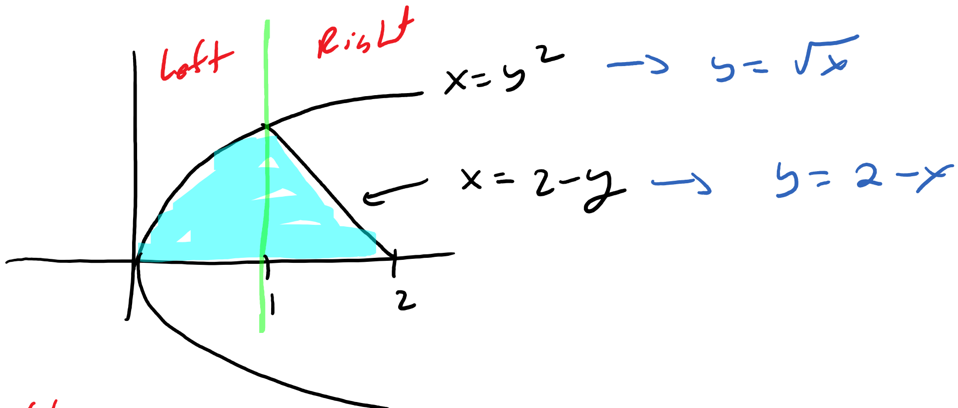
$x = 2-y$

$y=1 \rightarrow x=1$

$y=0 \rightarrow x=2$

Step 2 | Swap the direction of the red arrow. The new Integral will be a dy dx Integral.

notice that there will be 2 double integrals.



Left

$0 \le x \le 1$

$0 \le y \le \sqrt{x}$

Right

$1 \le x \le 2$

$0 \le y \le 2-x$

$$\int_{y=0}^{1} \int_{x=y^2}^{2-y} f(x,y)\,dx\,dy = \int_{x=0}^{1} \int_{y=0}^{\sqrt{x}} f(x,y)\,dy\,dx + \int_{x=1}^{2} \int_{y=0}^{2-x} f(x,y)\,dy\,dx$$

$y=0$ $x=y^2$

$x=0$ $y=0$ $x=1$ $y=0$

Problem 5

5. Evaluate the integral $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\, dx dy$.

we would need an x^2 in front of the square root in order to Integrate with respect to x.

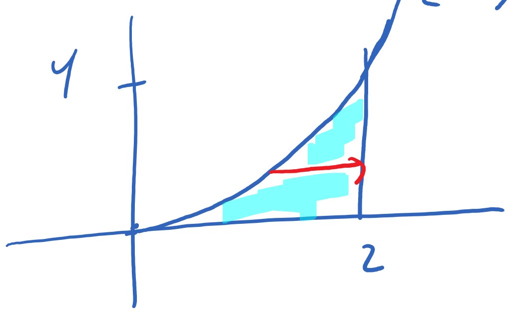
as It is written, This problem can not be done.

Change the order of the integration.

$\leftarrow x=\sqrt{y}$ or $y=x^2$

$0 \le y \le 4$

$\sqrt{y} \le x \le 2$



$0 \le x \le 2$

$0 \le y \le x^2$

$$\int_{y=0}^{4} \int_{x=\sqrt{y}}^{2} \sqrt{x^3+1}\, dx dy = \int_{x=0}^{2} \int_{y=0}^{x^2} \sqrt{x^3+1}\, dy\, dx$$

$$= \int_{x=0}^{2} y\sqrt{x^3+1}\, \Big|_{y=0}^{x^2} dx = \int_{x=0}^{2} x^2\sqrt{x^3+1}\, dx$$

$$\int_{x=0} \qquad \int_{y=0} \qquad \int_{x=0}$$

u-sub.
$u = x^3+1$

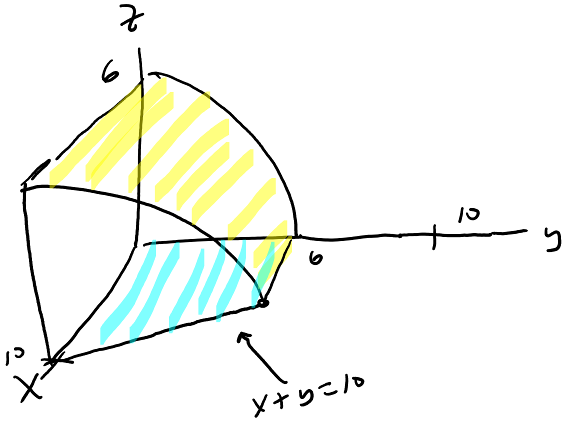
$$= \frac{1}{3}\cdot\frac{2}{3}\left(x^3+1\right)^{3/2}\Bigg|_{x=0}^{2}$$

$$= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\left(0+1\right)^{3/2} = \frac{2}{9}\left(9\right)^{3/2} - \frac{2}{9}$$

$$= \frac{2}{9}(27) - \frac{2}{9} = 6 - \frac{2}{9} = \frac{54}{9} - \frac{2}{9} = \frac{52}{9}$$

Problem 6

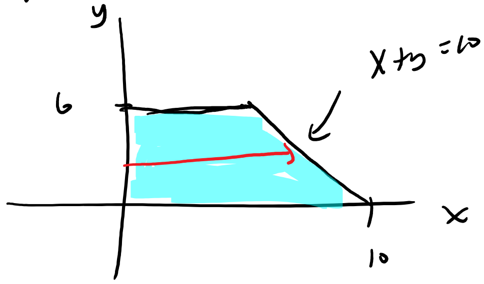
6. Find the volume of the solid bounded by the cylinder $y^2+z^2=36$, the planes $x=0$, $y=0$, $z=0$, $x+y=10$ in the first octant.



Top of the solid is the yellow part. This is: $z = +\sqrt{36-y^2}$

Bottom of the solid is the blue part. This is Region D

graph of D



↑ dy dx method is 2 double Integrals.

→ dx dy method is 1 double Integral

Lets use the dx dy method.

$0 \le y \le 6$

$0 \le x \le 10-y$

$$V = \iint_D \sqrt{36-y^2}\, dA = \int_{y=0}^{6} \int_{x=0}^{10-y} \sqrt{36-y^2}\, dx\, dy$$

$$= \int_{y=0}^{6} x\sqrt{36-y^2} \Big|_{x=0}^{10-y} dy$$

$$= \int_{y=0}^{6} (10-y)\sqrt{36-y^2}\, dy = \int^{6} 10\sqrt{36-y^2} - y\sqrt{36-y^2}\, dy$$

$$= \int_{y=0} (10-y)\sqrt{36-y^2}\, dy = \int_{y=0} \ldots$$

$$= \int_{y=0}^{6} \underbrace{10\sqrt{36-y^2}}_{\text{Trig Sub.}}\, dy - \int_{y=0}^{6} \underbrace{y\sqrt{36-y^2}}_{\text{u-sub.}}\, dy$$

Trig Sub. $y = 6\sin\theta$

u-sub. $u = 36 - y^2$

$\boxed{\text{note symbol lab would be nice to use Right now}}$

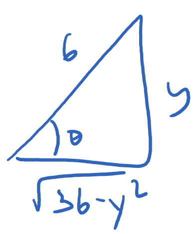
1st integral by hand

$y = 6\sin\theta$

$dy = 6\cos\theta\, d\theta$

$$\int 10\sqrt{36-y^2}\, dy = \int 10\sqrt{36-36\sin^2\theta}\cdot 6\cos\theta\, d\theta$$

$$= \int 60\cos\theta\cdot\sqrt{36\cos^2\theta}\, d\theta = \int 360\cos\theta\cdot\cos\theta\, d\theta$$



$\frac{y}{6} = \sin\theta$

$$= \int 360\cos^2\theta\, d\theta = 360\int \tfrac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= 180\left[\theta + \tfrac{1}{2}\sin 2\theta\right] = 180\left[\theta + \tfrac{1}{2}\cdot 2\sin\theta\cos\theta\right]$$

$$= 180\left[\arcsin\left(\frac{y}{6}\right) + \frac{y}{6}\cdot\frac{\sqrt{36-y^2}}{6}\right]$$

$$\int y\sqrt{36-y^2}\, dy = -\frac{1}{2}\cdot\frac{2}{3}(36-y^2)^{3/2} = -\frac{1}{3}(36-y^2)^{3/2}$$

u-sub $u = 36 - y^2$

$$\int^{6} 10\sqrt{36-y^2}\, dy - \int^{6} y\sqrt{36-y^2}\, dy$$

$$\int_{y=0}^{6} 10\sqrt{36-y^2}\,dy - \int_{y=0}^{6} y\sqrt{36-y^2}\,dy$$

$$= 180\left[\arcsin\left(\frac{y}{6}\right) + \frac{y}{6}\cdot\frac{\sqrt{36-y^2}}{6}\right] - -\frac{1}{3}\left(36-y^2\right)^{3/2}\Bigg|_0^6$$

$$= 180\left[\arcsin(1) + 0\right] + \frac{1}{3}(0) - \left(180\left[0+0\right] + \frac{1}{3}(36)^{3/2}\right)$$

$$= 180\arcsin(1) - \frac{1}{3}(6)^3 = 180\left(\frac{\pi}{2}\right) - \frac{1}{3}\cdot 216$$

$$= \boxed{90\pi - 72}$$

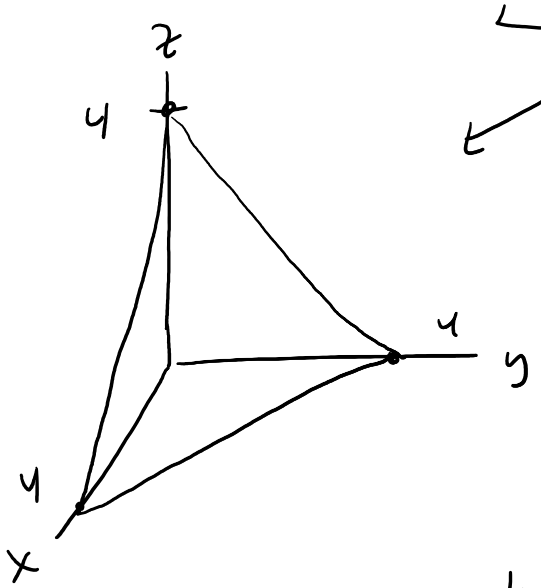
Problem 7

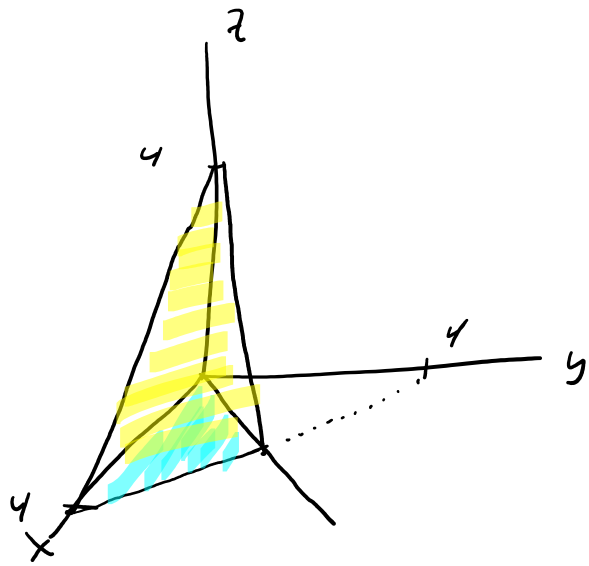
Solve for z

$z = 4 - x - y$

7. Setup the integral that would give the volumn of the solid (a tetrahedron) bounded by the planes $y = 0$, $z = 0$, $x = 3y$ and $x + y + z = 4$

graph of the plane. now lets look at the other parts.





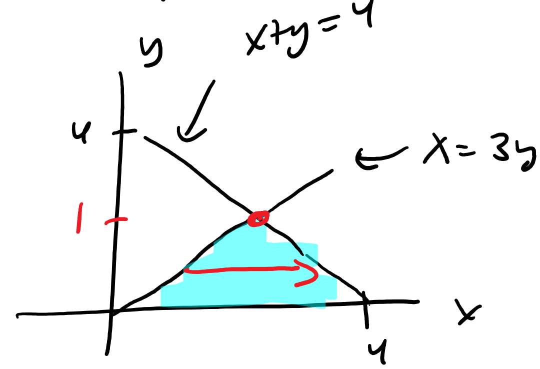
$y = 0 \to$ xz-plane

$z = 0 \to$ xy-plane

$x = 3y$

yellow part is the plane it is the top of the solid.

Blue part is the base, ie Region D.



$$V = \iint_D 4 - x - y \, dA$$

dx dy method

$0 \leq y \leq 1$

$3y \leq x \leq 4 - y$

Intersects

$x + y = 4$

$3y + y = 4$

$4y = 4$

$$\iint \quad \int_0^1 \int^{4-y}$$

$3y + y = 4$

$4y = 4$

$y = 1$

$$V = \iint_D 4 - x - y \, dA = \boxed{\int_{y=0}^{1} \int_{x=3y}^{4-y} 4 - x - y \, dx\, dy}$$