Problem 1

$x \quad y$

1. Evaluate the double integral where $R = [0,3] \times [1,5]$

$$\iint_R (x + 3y^2)\,dA = \int_{x=0}^{3} \int_{y=1}^{5} x + 3y^2 \, dy \, dx = \int_{x=0}^{3} (xy + y^3)\Big|_1^5 \, dx$$

$$= \int_0^3 5x + 125 - (x + 1) \, dx$$

$$= \int_0^3 4x + 124 \, dx = 2x^2 + 124x \Big|_0^3$$

$$= 18 + 372 - 0 = 390$$

Problem 2

2. If $R = [0, \ln(2)] \times [0, \ln(5)]$, evaluate

note $e^{2x-y} = e^{2x} e^{-y}$

$$\iint_R e^{2x-y}\,dA = \int_0^{\ln(2)} e^{2x}\,dx \cdot \int_0^{\ln(5)} e^{-y}\,dy \quad \text{by Fubini.}$$

$$= \frac{1}{2} e^{2x}\Big|_0^{\ln(2)} \cdot -e^{-y}\Big|_0^{\ln(5)}$$

$$= \frac{1}{2}\left(e^{2\ln(2)} - e^0\right) \cdot (-1)\left(e^{-\ln(5)} - e^0\right)$$

$$= \frac{1}{2}\left(e^{\ln(2^2)} - 1\right) \cdot (-1)\left(e^{\ln(5^{-1})} - 1\right)$$

$$= \frac{1}{2}(4 - 1) \cdot (-1)(5^{-1} - 1)$$

$$= \frac{1}{2}(3)(-1)\left(\frac{1}{5} - 1\right) = \left(-\frac{3}{2}\right)\left(-\frac{4}{5}\right) = \frac{12}{10}$$

Problem 3

3. Find the volume of the solid lying under the plane $\underbrace{2x + 6y - z + 1 = 0}$ on the region $R = [-1, 0] \times [1, 4]$

$\hookrightarrow 2x + 6y + 1 = z = f(x,y)$

$$V = \int_{x=-1}^{0} \int_{y=1}^{4} 2x + 6y + 1 \; dy \, dx = \int_{x=-1}^{0} \left(2xy + 3y^2 + y\right)\Big|_1^4 \, dx$$

$$= \int_{x=-1}^{0} (8x + 48 + 4) - (2x + 3 + 1) dx = \int_{x=-1}^{0} 6x + 48 \quad dx$$

$$= \left(3x^2 + 48x\right)\Big|_{-1}^{0} = 0 - (3 - 48)$$

$$= 0 - (-45) = 45$$

Problem 4

4. Evaluate the double integral where $R = [0,2] \times [0,1]$

$$\iint_R \frac{x}{1+xy}\,dA = \int_{x=0}^{2}\int_{y=0}^{1} \frac{x}{1+xy}\,dy\,dx = \int_{x=0}^{2} \ln(1+xy)\Big|_0^1 dx$$

$$= \int_{x=0}^{2} \ln(1+x) - \ln(1)\,dx = \int_{x=0}^{2} \ln(1+x)\,dx$$

now do Integration by parts.

D	I
$\ln(1+x)$	1
$\frac{1}{1+x}$	x

$$\int \ln(1+x)\,dx = x\ln(1+x) - \int \frac{x}{1+x}\,dx$$

$$= x\ln(1+x) - \int \frac{x+1-1}{1+x}\,dx$$

$$= x\ln(1+x) - \int 1 - \frac{1}{1+x}\,dx$$

$$= x\ln(1+x) - (x - \ln(1+x))$$

$$= x\ln(1+x) - x + \ln(1+x)$$

continue with the integral.

$$\int_0^2 \ln(1+x)\,dx = x\ln(1+x) - x + \ln(1+x)\Big|_0^2$$

$$= 2\ln(3) - 2 + \ln(3) - (0 - 0 + \ln(1))$$

$$= \underline{\underline{3\ln(3) - 2}}$$

Problem 5

5. Evaluate the double integral where $R = [0,2] \times [-4,4]$

$$\iint_R \frac{xy^2}{1+x^2}\,dA = \int_{x=0}^{2} \frac{x}{1+x^2}\,dx \cdot \int_{y=-4}^{4} y^2\,dy \qquad \text{by Fubini}$$

$$= \frac{1}{2}\ln|1+x^2|\Big|_0^2 \cdot \frac{1}{3}y^3\Big|_{-4}^{4}$$

$$= \frac{1}{2}\ln(5) \cdot \left(\frac{1}{3}4^3 - \frac{1}{3}(-4)^3\right)$$

$$= \frac{1}{2}\ln(5) \cdot \left(\frac{64}{3} - \frac{-64}{3}\right)$$

$$= \frac{1}{2}\ln(5) \cdot \frac{128}{3} = \boxed{\frac{128\ln(5)}{6}}$$