

Problem 1

Find all of the first order partial derivatives for $f(x, y) = (x^3 + y^2 e^{3x})^4$

$$f_x = 4(x^3 + y^2 e^{3x})^3 \cdot (3x^2 + 3y^2 e^{3x})$$

$$f_y = 4(x^3 + y^2 e^{3x})^3 \cdot 2y e^{3x}$$

Problem 2

Find all of the first order partial derivatives for $f(x, y, z) = \sin(5x)y^z + 4x^3y$.

$$f_x = 5 \cos(5x) y^z + 12x^2 y$$

$$f_y = \sin(5x) \cdot z y^{z-1} + 4x^3$$

$$f_z = \sin(5x) \cdot y^z \cdot \ln(y)$$

Problem 3

Use implicit differentiation to find $z_y = \frac{\partial z}{\partial y}$

$$x^4 y^3 + z^2 e^{2y} = 2y + \tan(4z)$$

$$\frac{\partial}{\partial y} (x^4 y^3 + z^2 e^{2y}) = \frac{\partial}{\partial y} (2y + \tan(4z))$$

$$3x^4 y^2 + 2z \frac{\partial z}{\partial y} e^{2y} + z^2 \cdot 2e^{2y} = 2 + 4\sec^2(4z) \frac{\partial z}{\partial y}$$

$$3x^4 y^2 + 2z^2 e^{2y} - 2 = -2z e^{2y} \frac{\partial z}{\partial y} + 4\sec^2(4z) \frac{\partial z}{\partial y}$$

$$3x^4 y^2 + 2z^2 e^{2y} - 2 = \left[-2z e^{2y} + 4\sec^2(4z) \right] \frac{\partial z}{\partial y}$$

$$\frac{3x^4 y^2 + 2z^2 e^{2y} - 2}{-2z e^{2y} + 4\sec^2(4z)} = \frac{\partial z}{\partial y}$$

Problem 4

Find z_x for $x^2 \sin(x^3 + y^2) + yz^2 = \cos(4z)$

$$\frac{\partial}{\partial x} (x^2 \sin(x^3 + y^2) + yz^2) = \frac{\partial}{\partial x} \cos(4z)$$

$$2x \sin(x^3 + y^2) + x^2 \cdot 3x^2 \cos(x^3 + y^2) + 2yz \frac{\partial z}{\partial x} = -4 \sin(4z) \frac{\partial z}{\partial x}$$

$$2x \sin(x^3 + y^2) + 3x^4 \cos(x^3 + y^2) = [-4 \sin(4z) - 2yz] \frac{\partial z}{\partial x}$$

$$\frac{2x \sin(x^3 + y^2) + 3x^4 \cos(x^3 + y^2)}{-4 \sin(4z) - 2yz} = \frac{\partial z}{\partial x}$$