Problem 1

Example: Given $\mathbf{r}(t) = \left\langle t\sqrt{t+5},\ t^2+2,\ \dfrac{e^t-1}{t} \right\rangle$

(a) Find the domain of $\mathbf{r}(t)$.

(b) Find all t where $\mathbf{r}(t)$ is continuous.

(c) Compute $\lim_{t\to 0} \mathbf{r}(t)$.

$x(t) = t\sqrt{t+5}$

$y(t) = t^2+2$

$z(t) = \dfrac{e^t-1}{t}$

A] for the $x(t)$ need $t \geq -5$

for the $y(t)$ all Real #s for t

for the $z(t)$ need $t \neq 0$

domain: $[-5, 0) \cup (0, \infty)$ ← Interval notation.

t is all Reals numbers such that $t \geq -5$ and $t \neq 0$

B] $r(t)$ is continuous for $[-5, 0) \cup (0, \infty)$

C] $\lim_{t\to 0} r(t) = \left\langle \lim_{t\to 0} x(t),\ \lim_{t\to 0} y(t),\ \lim_{t\to 0} z(t) \right\rangle$

$\lim_{t\to 0} x(t) = \lim_{t\to 0} t\sqrt{t+5} = 0$

$\lim_{t\to 0} y(t) = \lim_{t\to 0} t^2+2 = 2$

$$\lim_{t\to 0} z(t) = \lim_{t\to 0} \frac{e^t - 1}{t} \overset{L'H}{=} \lim_{t\to 0} \frac{e^t}{1} = e^0 = 1$$

$$\lim_{t\to 0} r(t) = \langle 0, 2, 1 \rangle$$

Problem 2

$x \quad y \quad z$

At what points does the the curve $\mathbf{r}(t) = \langle 2\sin t, 4t, \cos t\rangle$ intersect the ellipsesoid $x^2 + y^2 + 4z^2 = 10$? If there are none, expain why you know this.

Intersection points are points on the space curve that also satisfy the equation $x^2+y^2+4z^2=10$

$$(2\sin(t))^2 + (4t)^2 + 4(\cos t)^2 = 10$$

$$4\sin^2 t + 16t^2 + 4\cos^2 t = 10$$

$$16t^2 + 4[\sin^2 t + \cos^2 t] = 10$$

$$16t^2 + 4 = 10$$

$$16t^2 = 6$$

$$t^2 = \frac{6}{16}$$

$$t = \pm\sqrt{\frac{6}{16}} = \pm\frac{\sqrt{6}}{4}$$

yes it does intersect. now find the points.

$\boxed{t = \frac{\sqrt{6}}{4}}$ $\quad r\left(\frac{\sqrt{6}}{4}\right) = \left\langle 2\sin\left(\frac{\sqrt{6}}{4}\right), \sqrt{6}, \cos\left(\frac{\sqrt{6}}{4}\right)\right\rangle$

$\boxed{t = -\frac{\sqrt{6}}{4}}$ $\quad r\left(-\frac{\sqrt{6}}{4}\right) = \left\langle 2\sin\left(-\frac{\sqrt{6}}{4}\right), -\sqrt{6}, \cos\left(-\frac{\sqrt{6}}{4}\right)\right\rangle$

Problem 3

Find the points where the line through the points $(1,0,2)$ and $(5,-1,2)$ intersects the surface $x = y^2 + z^2$.

find the eq of the line.

$V = \langle 5-1, -1-0, 2-2 \rangle = \langle 4, -1, 0 \rangle$

$x = 1 + 4t$
$y = -t$
$z = 2$

now find the Intersection point.

$1+4t = (-t)^2 + (2)^2$
$1+4t = t^2 + 4$
$0 = t^2 - 4t + 3$
$0 = (t-3)(t-1)$
$t = 3, \quad t = 1$

$\boxed{t=1}$ $x = 5$, $y = -1$, $z = 2$

$\boxed{t=3}$ $x = 13$, $y = -3$, $z = 2$

Problem 4

Find a vector function that represents the curve of intersection of the two surfaces.

$x = y^2 - z^2$ $\qquad\qquad$ $y^2 + z^2 = 4$

There is more than one correct answer.

Let $y = 2\sin(t)$ $\quad$ Since $y^2 + z^2 = 4$ is a circle

$z = 2\cos(t)$ $\quad$ of Radius 2.

Then $\quad X = 4\sin^2(t) - 4\cos^2(t)$

$\boxed{\text{Answer}}$ $\quad r(t) = \langle 4\sin^2(t) - 4\cos^2(t),\ 2\sin t,\ 2\cos t \rangle$