Problem 1

Find the equation of the plane containing the point (1, 1, 1) and perpendicular to the line

$$\underbrace{\frac{x-1}{2} = \frac{y+2}{5} = \frac{1-z}{3}} = t$$

find parametric equations (set equal to t and solve for x, y, + z)

Be careful on this part

$X = 1 + 2t$

$y = -2 + 5t$

$z = 1 - 3t$

direction vector of the line $v = \langle 2, 5, -3 \rangle$

Line is perp. to plane so v is a normal vector

Answer $2(x-1) + 5(y-1) - 3(z-1) = 0$

Problem 2

Show that these lines are skew.

Line 1:	Line 2
$x = 3 + t$	$x = 4 - v$
$y = 2 - 4t$	$y = 3 + v$
$z = t$	$z = -2 + 3v$

direction vectors

$v_1 = \langle 1, -4, 1 \rangle$

$v_2 = \langle -1, 1, 3 \rangle$

There is no value of c such that $v_1 = c v_2$ so the lines are not parallel.

Solve for Intersection

$x_1 = x_2$

$3 + t = 4 - v$

$\underline{v = 1 - t}$

$v = \frac{5}{3}$

$y_1 = y_2$

$2 - 4t = 3 + v$

$2 - 4t = 3 + 1 - t$

$-2 = 3t$

$t = \frac{-2}{3}$

now check z component.

Line 1

$z = t \rightarrow z = \frac{-2}{3}$

Line 2

$z = -2 + 3v \rightarrow z = -2 + 3\left(\frac{5}{3}\right)$

$z = -2 + 5$

$z = 3$

Since the z values are not the same, the lines will be skew.

Problem 3

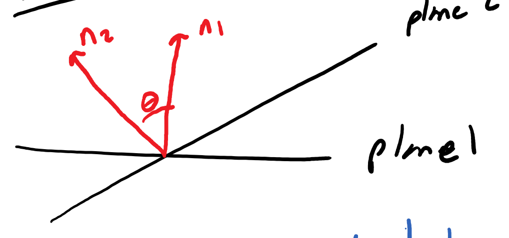
Find the angle between these planes.

$x + 2y + z = 4 \qquad 3x + 6y + 2z = 12$

$n_1 = \langle 1, 2, 1 \rangle \qquad n_2 = \langle 3, 6, 2 \rangle$

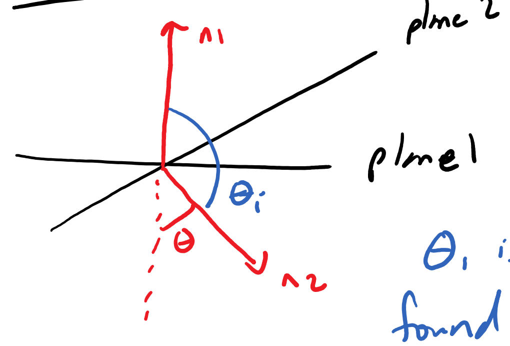
The angle between 2 non-parallel planes is the acute angle ($0 < \theta < \frac{\pi}{2}$)

Case 1



use dot product to find θ

Case 2



θ_1 is the angle found from the dot product. Value is larger than $\frac{\pi}{2}$ so adjust for the final answer.

$$n_1 \cdot n_2 = |n_1||n_2| \cos\theta$$

$$3 + 12 + 2 = \sqrt{6}\sqrt{49} \cos\theta$$

$$\frac{17}{7\sqrt{6}} = \cos\theta$$

$\theta = 7.49$ degrees OR 0.1307 Radians

Problem 4

Determine if these lines are parallel. If you answer was no, then determine if the lines are intersecting or skew. Justify your answer.

$L_1: \quad x = t + 2,\ y = 1 + 4t,\ z = 2t$

$L_2: \quad \frac{x-1}{2} = \frac{y-5}{4} = m;\ z = 10$

$V_1 = \langle 1, 4, 2 \rangle$

$V_2 = \langle 2, 4, 0 \rangle$

vectors are not parallel.

$x = 1 + 2m$

$y = 5 + 4m$

$z = 10$

Check for Intersection.

$x_1 = x_2$

$t + 2 = 1 + 2m$

$5 + 2 = 1 + 2m$

$6 = 2m$

$m = 3$

$z_1 = z_2$

$2t = 10$

$t = 5$

check y-value

$y_1 = 1 + 4t$ @ $t=5$

$y_1 = 21$

$y_2 = 5 + 4m$ @ $m=3$

$y_2 = 5 + 12 = 17$

Since the y-values are not the same when x & z values are equal and the lines are not parallel

the lines are skew.

Problem 5

Does the line L lie in a plane that would be parallel to the plane P? Justify your answer.

L: $x = 1 + 3t$, $y = 1 + t$, $z = 1 - 5t$ $\quad v = \langle 3, 1, -5 \rangle$

P: $x + 2y + z = 5$ $\quad n = \langle 1, 2, 1 \rangle$

Does L Intersect P? (ie. Is the line on the plane)

$1 + 3t + 2(1+t) + (1-5t) = 5$

$1 + 3t + 2 + 2t + 1 - 5t = 5$

$4 = 5 \leftarrow$ not valid so line doesn't Intersect.

Are V & n perp?

$v \cdot n = \langle 3, 1, -5 \rangle \cdot \langle 1, 2, 1 \rangle$

$= 3 + 2 - 5$

$= 0$

vectors are perpendicular.

Since L is not on the plane P, then there exist a plane P_2 that is parallel to P and contains L.

Problem 6

Does the line L lie in a plane that would be parallel to the plane P? Justify your answer.

L: $x = 1 + 4t$, $y = 1 + 2t$, $z = 1 - t$

$V = \langle 4, 2, -1 \rangle$

P: $x + 3y + z = 23$

$n = \langle 1, 3, 1 \rangle$

Does L Intersect P ?

$$1+4t + 3(1+2t) + 1-t = 23$$
$$1+4t + 3+6t + 1-t = 23$$
$$9t + 5 = 23$$
$$9t = 18$$
$$t = 2$$

The line Intersects the plane at $t=2$ so L can not be in a plane that is parallel to P.

method 2)

$$V \cdot n = \langle 4, 2, -1 \rangle \cdot \langle 1, 3, 1 \rangle$$
$$= 4 + 6 - 1$$
$$= 9$$

for L to be in a plane parallel to P means The normal vector for P and the directional vector of L would need to be perpendicular. (and the line could not be on P)

Since $n \cdot v = 9 \neq 0$ Then L is not in a plane That is parallel to P.

Problem 7

Find the distance from the line from the plane.

Line: $x = 1 - 3t$, $y = 1 + t$, $z = 1 + t$

$v = \langle -3, 1, 1 \rangle$

Plane: $x + 2y + z = 10$ $\rightarrow$ $x + 2y + z - 10 = 0$

$n = \langle 1, 2, 1 \rangle$

$v \cdot n = -3 + 2 + 1 = 0$

Since v and n are perpendicular we know the line is either on P or is parallel to P.

ie distance $= 0$

pick a point on the line (let $t=0$) point $(1,1,1)$

$$\text{distance} = \frac{|1 + 2(1) + 1 - 10|}{\sqrt{1^2 + 2^2 + 1^2}} = \frac{|4-10|}{\sqrt{6}} = \frac{6}{\sqrt{6}}$$

Problem 8

Find the distance from the line from the plane.

Line: $x = 1 + 2t$, $y = 1 + t$, $z = 1 + t$ $\qquad v = \langle 2, 1, 1 \rangle$

Plane: $x + 2y + z = 10$ $\qquad n = \langle 1, 2, 1 \rangle$

$n \cdot v = 2 + 2 + 1 = 5$ These vectors are not perp. so that means the line will intersect the plane. distance is zero due to Intersection

method 2 for showing Intersection of the line + plane

$$(1 + 2t) + 2(1 + t) + (1 + t) = 10$$

$$1 + 2t + 2 + 2t + 1 + t = 10$$

$$5t + 4 = 10$$

$$5t = 6$$

$$t = 6/5$$

This shows The line Intersects the plane when $t = \frac{6}{5}$