Problem 1

If $\mathbf{a} = \langle 6, 0, -2 \rangle$, find a vector $\mathbf{b}$ such that $\text{comp}_{\mathbf{a}}\mathbf{b} = 3$

Let $b = \langle x, y, z \rangle$

$$\text{comp}_a b = \frac{a \cdot b}{|a|} = \frac{6x - 2z}{\sqrt{36 + 0 + 4}} = \frac{6x - 2z}{\sqrt{40}} = 3$$

So $6x - 2z = 3\sqrt{40}$

now pick a # for z Let $z = 0$

Then $x = \frac{3\sqrt{40}}{6} = \frac{\sqrt{40}}{2} = \frac{2\sqrt{10}}{2} = \sqrt{10}$

Since y is not part of this equation then we can let y be any # so Let $y = 5$

Thus $b = \langle \sqrt{10}, 5, 0 \rangle$

Problem 2

Find a vector of length 5 that has the following directional angles: $\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{6}$, and $\gamma = \frac{\pi}{2}$,

Let $a = \langle x, y, z \rangle$ and $|a| = 5$

$\cos\alpha = \frac{x}{|a|} \Rightarrow \cos\left(\frac{\pi}{3}\right) = \frac{x}{5} \Rightarrow x = 5\cos\left(\frac{\pi}{3}\right)$

$x = 5 \cdot \frac{1}{2}$

$\cos\beta = \frac{y}{|a|} \Rightarrow \cos\left(\frac{\pi}{6}\right) = \frac{y}{5} \Rightarrow y = 5\cos\left(\frac{\pi}{6}\right)$

$y = \frac{5\sqrt{3}}{2}$

$\cos\gamma = \frac{z}{|a|} \Rightarrow \cos\left(\frac{\pi}{2}\right) = \frac{z}{5} \Rightarrow 0 = z$

$a = \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2}, 0 \right\rangle$