

3) Find a power series for the function $f(x)$.

$$f(x) = \frac{x}{(1+2x^3)^2}$$

Known Fact:

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

with $|x| < 1$

Consider $g(x) = \frac{1}{1+2x^3}$

$$g'(x) = \frac{-6x^2}{(1+2x^3)^2} = -6x \cdot \frac{x}{(1+2x^3)^2}$$

Then $f(x) = \frac{-1}{6x} g'(x)$

Now compute the series

$$g(x) = \frac{1}{1 - (-2x^3)} = \sum_{n=0}^{\infty} (-2x^3)^n = \sum_{n=0}^{\infty} (-1)^n 2^n x^{3n}$$

$$g'(x) = \sum_{n=1}^{\infty} (-1)^n 2^n (3n) x^{3n-1}$$

$$\text{Thus } f(x) = \frac{-1}{6x} \sum_{n=1}^{\infty} (-1)^n 2^n (3n) x^{3n-1}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^n (3n) x^{3n-1}}{6x} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2 \cdot 2^{n-1} \cdot 3n x^{3n-1}}{6x}$$

$$f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} 2^{n-1} n x^{3n-2}$$

1) Find a power series for the function $f(x)$.