Example: Determine if the series is absolute convergent, convergent, or divergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+1}$$

$$a_n = \frac{(-1)^n \sqrt{n}}{n+1} \qquad a_{n+1} = \frac{(-1)^{n+1} \sqrt{n+1}}{n+2}$$

ratio test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \underbrace{\frac{\sqrt{n+1}}{n+2}}_{a_{n+1}} \cdot \underbrace{\frac{n+1}{\sqrt{n}}}_{\frac{1}{a_n}} = \lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n}} \cdot \frac{n+1}{n+2}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n+1}{n}} \cdot \frac{n+1}{n+2} = \sqrt{1} \cdot 1 = 1$$

The ratio test did not give any information. ∴ Try something else.

Example: The series $\sum a_n$ is defined recursively by

$$a_1 = 1 \qquad a_{n+1} = \left[\frac{(2+\cos(n))}{\sqrt{n}}\right] a_n \text{ for } n \geq 1.$$

Is the series is absolute convergent, convergent, or divergent?

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{2+\cos(n)}{\sqrt{n}} \cancel{a_n}}{\cancel{a_n}} \right| = \lim_{n\to\infty} \left| \frac{2+\cos(n)}{\sqrt{n}} \right| = 0$$

The series is abs. conv.