3) Does this series converge or diverge?

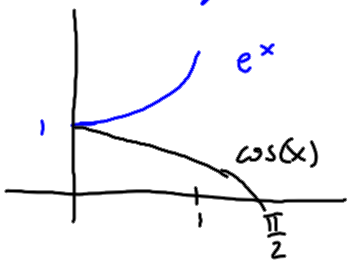
$$\sum_{n=1}^{\infty} e^{1/n} - \cos\left(\frac{1}{n}\right)$$

Note $\lim_{n\to\infty} e^{1/n} - \cos\left(\frac{1}{n}\right) = 1 - 1 = 0$

note $0 < \frac{1}{n} \leq 1$ for $n \geq 1$

So the values of $e^{1/n}$ are the same values of e^x on the interval $(0,1]$. Likewise for $\cos(\frac{1}{n})$

from the graphs we see that $e^{1/n} - \cos(\frac{1}{n})$ will always be positive. Thus we



can use the Limit comparison Test.

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compare with $\sum \frac{1}{n} \to$ divergent

$$\lim_{n\to\infty} \frac{e^{1/n} - \cos\left(\frac{1}{n}\right)}{\frac{1}{n}} \overset{\frac{0}{0}}{=} \text{ use L'Hopital's}$$

$$\lim_{n\to\infty} \frac{\frac{-1}{n^2}\cdot e^{1/n} - -\sin\left(\frac{1}{n}\right)\cdot\frac{-1}{n^2}}{\frac{-1}{n^2}}$$

$$= \lim_{n\to\infty} \frac{\frac{-1}{n^2}\left[e^{1/n} + \sin\left(\frac{1}{n}\right)\right)}{\frac{-1}{n^2}}$$

$$= \lim_{n\to\infty} e^{1/n} + \sin\left(\frac{1}{n}\right) = 1 + 0 = 1$$

By LCT both series diverge.

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