11.3 #2)

A) $f(x) = x e^{-x^2/2}$ The series is positive, continuous and decreasing.

$$f' = 1 e^{-x^2/2} + x \cdot \left(\frac{-2x}{2}\right) e^{-x^2/2}$$

$$= 1 e^{-x^2/2} - x^2 e^{-x^2/2}$$

$$= (1-x^2)\, e^{-x^2/2}$$

note $e^{-x^2/2}$ is positive and $(1-x^2) < 0$ for $x > 1$

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now check the integral

$$\int_1^{\infty} x e^{-x^2/2}\, dx \qquad u = \frac{-x^2}{2} \qquad du = \frac{-2x}{2}\, dx$$

$du = -x\,dx$

$-du = x\,dx$

$$= \lim_{t \to \infty} \int_{x=1}^{x=t} -e^u\, du$$

$$= \lim_{t \to \infty} -e^u \Big|_{x=1}^{x=t} = \lim_{t \to \infty} -e^{-x^2/2} \Big|_1^t$$

$$= \lim_{t \to \infty} -e^{-t^2/2} - -e^{-1/2} = 0 + e^{-1/2}$$

The integral converges

By the Integral test the series will converge.

B] $S_4 = 1e^{-1/2} + 2e^{-4/2} + 3e^{-9/2} + 4e^{-16/2}$

$$R_4 \leq \int_4^{\infty} x e^{-x^2/2}\, dx = e^{-16/2} = e^{-8}$$

Integral work from part A