11.3

#1 Integral test.

$f(x) = \dfrac{1}{x^2+3x+2}$ is positive, continuous for $x>0$. and decreasing. for $x>0$

$f' = -1(x^2+3x+2)^{-2}(2x+3) = \dfrac{-(2x+3)}{(x^2+3x+2)^2} < 0$

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determine if $\displaystyle\int_3^\infty \frac{1}{x^2+3x+2}\,dx$ converges.

method 1 comparison method.

$x^2+3x+2 > x^2 \Rightarrow \dfrac{1}{x^2+3x+2} < \dfrac{1}{x^2}$

$\displaystyle\int_3^\infty \frac{1}{t^2}\,dx$ is a p-integral so it converges.

by the integral test The series will converge.

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method 2: compute the integral.

$\dfrac{1}{x^2+3x+2} = \dfrac{A}{x+1} + \dfrac{B}{x+2} \Rightarrow 1 = A(x+2) + B(x+1)$

if $x=-1$ $\quad 1 = A(1)$ $\quad A=1$ $\qquad x=-2$ $\quad 1=-B$ $\quad B=-1$

$$\int_3^\infty \frac{1}{x^2+3x+2}\,dx = \int_3^\infty \frac{1}{x+1} - \frac{1}{x+2}\,dx$$

$$= \lim_{t\to\infty} \int_3^t \frac{1}{x+1} - \frac{1}{x+2}\,dx$$

$$= \lim_{t\to\infty} \Big[\ln|x+1| - \ln|x+2| \Big]_3^t$$

$$= \lim_{t\to\infty} \ln(t+1) - \ln(t+2) - (\ln(4) - \ln(5))$$

$$= \lim_{t\to\infty} \ln\left(\frac{t+1}{t+2}\right) - \ln\left(\frac{4}{5}\right)$$

$$= \ln(1) - \ln\left(\frac{4}{5}\right)$$

Thus the integral converges.

By the Integral test the series will converge