2) Use a Maclaurin series to approximate this integral to 4 decimal places. i.e. error < 0.00005

$$\int_0^{1/2} \frac{\ln(1+x)}{x}\,dx$$

step 1: find the series for ln(1+x).

$$y = \ln(1+x)$$

$$y' = \frac{1}{1+x} = \sum_{n=0} (-1)^n x^n$$

$$y = \ln(1+x) = C + \int \sum_{n=0} (-1)^n x^n = C + \sum_{n=0} \frac{(-1)^n x^{n+1}}{n+1}$$

when $x=0$ we get

$$\ln(1+0) = C + \sum 0$$

$$\ln(1) = C$$

$$0 = C$$

Thus $\ln(1+x) = \sum_{n=0} \frac{(-1)^n x^{n+1}}{n+1}$

Step 2: find the series for $\frac{\ln(1+x)}{x}$

$$\frac{\ln(1+x)}{x} = \sum_{n=0} \frac{(-1)^n x^n}{n+1}$$

Step 3: now evaluate the integral

$$\int_0^{1/2} \frac{\ln(1+x)}{x}\,dx = \int_0^{1/2} \sum_{n=0} \frac{(-1)^n x^n}{n+1}\,dx = \sum_{n=0} \frac{(-1)^n x^{n+1}}{(n+1)^2}\Bigg|_0^{1/2}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{2}\right)^{n+1}}{(n+1)^2} - \sum 0$$

$$= \sum_{n=0} \frac{(-1)^n}{(n+1)^2\, 2^{n+1}}$$

Last step. now use the remainder information for the alternating serries to approximate the summation so that the error is < 0.00005.

$$b_n = \frac{1}{(n+1)^2\, 2^{n+1}}$$

$b_0 = .5$

$b_1 = .0625$

$b_2 \quad .0138889$

$b_3 = .00390625$

$b_4 = .00125$

$b_5 = .000434$

$b_6 = .00015943$

$b_7 = .000061035l6$

$b_8 = .0000241l$

$b_8 <$ error $= .00005$

$$\int_0^{1/2} \frac{\ln(1+x)}{x}\,dx = \sum_{n=0} \frac{(-1)^n}{(n+1)^2\, 2^{n+1}}$$

$$\approx b_0 - b_1 + b_2 - b_3 + b_4 - b_5 + b_6 - b_7$$

note: the sign on the term b 0is positive since when n=0 the (-1) nis a 1.