Section 7.2: Additional Problems Solutions

1. $\displaystyle\int \cot^6(3x)\csc^4(3x)\ dx = \int \cot^6(3x)\csc^2(3x)\csc^2(3x)\ dx = \int \cot^6(3x)(1+\cot^2(3x))\csc^2(3x)\ dx$

   at this point if you integrate directly you have the following work.

   $$= \int \cot^6(3x)\csc^2(3x) + \cot^8(3x)\csc^2(3x)\ dx = \frac{-1}{7}\cdot\frac{1}{3}\cot^7(3x) + \frac{-1}{9}\cdot\frac{1}{3}\cot^9(3x)$$
   $$= \frac{-1}{21}\cot^7(3x) + \frac{-1}{27}\cot^9(3x) + C$$

   you may also do a $u-$sub

   $u = \cot(3x)$ gives $du = -3\csc^2(3x)dx$ or $\dfrac{-1}{3}du = \csc^2(3x)dx$

   $$\int \cot^6(3x)(1+\cot^2(3x))\csc^2(3x)\ dx = \int \frac{-1}{3}u^6(1+u^2)du = \frac{-1}{3}\int u^6 + u^8 du$$
   $$= \frac{-1}{3}\cdot\left(\frac{u^7}{7} + \frac{u^8}{8}\right) + C = \frac{-1}{21}\cot^7(3x) + \frac{-1}{27}\cot^9(3x) + C$$

2. $\displaystyle\int \tan^2(x)\sec(x)\ dx = \int (\sec^2(x) - 1)\sec(x)dx = \int \sec^3(x) - \sec(x)dx$

   Now use the formulas for $\int sec(x)dx$ and $\int sec^3(x)dx$ that were given in class.

   $$= \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| - \ln|\sec x + \tan x| + C$$
   $$= \frac{1}{2}\sec x\tan x - \frac{1}{2}\ln|\sec x + \tan x| + C$$

3. $\displaystyle\int \tan^4(x)\ dx = \int \tan^2(x)\tan^2(x)dx = \int \tan^2(x)\cdot(\sec^2(x) - 1)dx = \int \tan^2(x)\sec^2(x) - \tan^2(x)dx$

   $$= \int \tan^2(x)\sec^2(x) - (\sec^2(x) - 1)dx = \int \tan^2(x)\sec^2(x) - \sec^2(x) + 1dx$$
   $$= \int \tan^2(x)\sec^2(x)dx - \int \sec^2(x)dx + \int 1dx$$

   Solve the first integral using a $u-$sub of $u = \tan(x)$. The last two integral are just regular integrals.

   Answer: $= \dfrac{1}{3}\tan^3(x) - \tan(x) + x + C$

4. $\displaystyle\int \tan^3(x)\ dx = \int \tan^2(x)\tan(x)dx = \int (\sec^2(x)-1)\tan(x)dx = \int \sec^2(x)\tan(x)dx - \int \tan(x)dx$

Now we do both integrals with a $u-$sub.

$$\int \sec^2(x)\tan(x)dx = \int \sec(x)\sec(x)\tan(x)dx = \int u du = \frac{u^2}{2} = \frac{1}{2}\sec^2(x)$$

$u = \sec(x)$ and $du = \sec(x)\tan(x)dx$

$$\int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)}dx = \int \frac{-1}{u}du = -\ln|u| = -\ln|\cos(x)|$$

where $u = \cos(x)$ and $du = -\sin(x)dx$

Answer: $\frac{1}{2}\sec^2(x) + \ln|\cos(x)| + C$

5. $\displaystyle\int x\sin^2(x)\ dx = \int x\cdot\frac{1}{2}(1-\cos(2x))dx = \int \frac{x}{2} - \frac{x}{2}\cos(2x)dx = \int \frac{x}{2}dx - \int \frac{x}{2}\cos(2x)dx$

$$= \frac{x^2}{4} - \int \frac{x}{2}\cos(2x)dx$$

Now use integration by parts to solve this integral.

$$\int \frac{x}{2}\cos(2x)dx = \frac{x}{4}\sin(2x) + \frac{1}{8}\cos(2x) + C$$

Derivative		Integral
$\frac{x}{2}$	+	$\cos(2x)$
$\frac{1}{2}$	−	$\frac{1}{2}\sin(2x)$
0	$+\int$	$\frac{-1}{4}\cos(2x)$

Answer:

$$\int x\sin^2(x)\ dx = \frac{x^2}{4} - \left(\frac{x}{4}\sin(2x) + \frac{1}{8}\cos(2x)\right) + C$$
$$= \frac{x^2}{4} - \frac{x}{4}\sin(2x) - \frac{1}{8}\cos(2x) + C$$

6. $\displaystyle\int \sin^2(x)\cos^2(x)\ dx = \int \frac{1}{2}(1-\cos(2x)\cdot\frac{1}{2}(1+\cos(2x)dx = \frac{1}{4}\int 1-\cos^2(2x)dx$

$$= \frac{1}{4}\int 1 - \frac{1}{2}(1+\cos(4x)dx = \frac{1}{4}\int 1 - \frac{1}{2} - \frac{1}{2}\cos(4x)dx = \frac{1}{4}\int \frac{1}{2} - \frac{1}{2}\cos(4x)dx$$

$$= \frac{1}{4}\left(\frac{x}{2} - \frac{1}{8}\sin(4x)\right) + C = \frac{x}{8} - \frac{1}{32}\sin(4x) + C$$