#2

2) Here are two lines represented by the vector equations, L_1 and L_2.

$$L_1(t) = \langle 1+t, 8+3t \rangle \qquad L_2(s) = \langle 3-s, 7-2s \rangle$$

A) Determine if these lines are parallel, perpendicular, or neither.
B) If the lines are not parallel, then find the angle θ, where $0 < \theta \leq \frac{\pi}{2}$, that is made at the intersection of the two lines.

$L_1(t) = \langle 1, 8 \rangle + t\langle 1, 3 \rangle \longrightarrow V_1 = \langle 1, 3 \rangle$

$L_2(s) = \langle 3, 7 \rangle + s\langle -1, -2 \rangle \qquad V_2 = \langle -1, -2 \rangle$

Since there is not a value of c such that $V_1 = cV_2$, L_1 and L_2 are not parallel.

Since $V_1 \cdot V_2 = -1 - 6 \neq 0$ the lines are not perpendicular.

$V_1 \cdot V_2 = -7 = |V_1||V_2| \cos\theta$

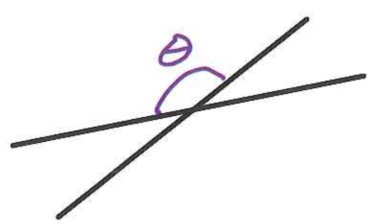
$|V_1| = \sqrt{1+9} = \sqrt{10}$

$|V_2| = \sqrt{1+4} = \sqrt{5}$

$-7 = \sqrt{10}\sqrt{5}\cos\theta$

$\theta = 171.87^\circ$

The graph of the lines looks something similar to the picture on the left. The dot product formula used above will give the angle theta where theta is from 0 to pi(180 degrees). The problem is asking for the angle between the lines to be between 0 and pi/2 (90 degrees).



Answer $= 180 - 171.87 = 8.13^\circ$

part c)

Since the lines are not parallel, they have at least one point in common. to solve for this we set the components equal.

x components	y components
$1+t = 3-s$	$8+3t = 7-2s$

we get

$t+s = 2$ $\quad\quad$ $3t + 2s = -1$

$s = 2-t$

$3t + 2(2-t) = -1$

$3t + 4 - 2t = -1$

$t = -5$

$s = 2 - -5$

$s = 7$

Since there is only one solution for s & t we know the lines intersect at only one point, and that

$L_1(-5) = L_2(7)$. $\quad\quad$ $L_1(-5) = \langle -4, -7 \rangle$

The point of intersection is $(-4, -7)$