Section 2.2: Additional Problems Solutions

2. Evaluate these limits.

   (a) If you plug in $x = 5$ into $f(x) = \dfrac{1}{x-5}$, you get the form $\frac{\text{non−zero number}}{0}$. This means that $x = 5$ is a vertical asymptote.

   Now pick a number slightly larger than $x = 5$, say 5.1, since the limit is from the right and evaluate $f(x)$. Notice that both the numerator and the denominator are positive.

   Thus $\displaystyle\lim_{x\to 5^+} \frac{1}{x-5} = +\infty$

   (b) If you plug in $x = 5$ into $f(x) = \dfrac{1}{x-5}$, you get the form $\frac{\text{non−zero number}}{0}$. This means that $x = 5$ is a vertical asymptote.

   Now pick a number slightly smaller than $x = 5$, say 4.9, since the limit is from the left and evaluate $f(x)$. Notice that the numerator is positive and the denominator is negative.

   Thus $\displaystyle\lim_{x\to 5^-} \frac{1}{x-5} = -\infty$

   (c) $\displaystyle\lim_{x\to 5} \frac{1}{x-5} = DNE$ since the left and right limits are not equal.

   (d) If you plug in $x = 0$ into $f(x) = \dfrac{x-5}{x^2}$, you get the form $\frac{\text{non−zero number}}{0}$. This means that $x = 0$ is a vertical asymptote.

   Now pick a number slightly larger than $x = 0$, say 0.1, since the limit is from the right and evaluate $f(x)$. Notice that the numerator is negative and the denominator is positive.

   Thus $\displaystyle\lim_{x\to 0^+} \frac{x-5}{x^2} = -\infty$

   Now pick a number slightly smaller than $x = 0$, say -0.1, since the limit is from the left and evaluate $f(x)$. Notice that the numerator is negative and the denominator is postive.

   Thus $\displaystyle\lim_{x\to 0^-} \frac{x-5}{x^2} = -\infty$

   Thus $\displaystyle\lim_{x\to 0} \frac{x-5}{x^2} = -\infty$ since both left and right limits agree.

   (e) If you plug in $x = 4$ into $f(x) = \dfrac{2-x}{4-x}$, you get the form $\frac{\text{non−zero number}}{0}$. This means that $x = 4$ is a vertical asymptote.

   Now pick a number slightly larger than $x = 4$, say 4.1, since the limit is from the right and evaluate $f(x)$. Notice that both the numerator and the denominator are negative.

   Thus $\displaystyle\lim_{x\to 4^+} \frac{2-x}{4-x} = +\infty$

   (f) If you plug in $x = 4$ into $f(x) = \dfrac{2-x}{4-x}$, you get the form $\frac{\text{non−zero number}}{0}$. This means that $x = 4$ is a vertical asymptote.

   Now pick a number slightly smaller than $x = 4$, say 3.9, since the limit is from the left and evaluate $f(x)$. Notice that the numerator is negative and the denominator is positive.

   $\displaystyle\lim_{x\to 4^-} \frac{2-x}{4-x} = -\infty$

(g) If you plug in $x = 3$ into $f(x) = \dfrac{x-2}{x^2-9}$, you get the form $\frac{\text{non-zero number}}{0}$. This means that $x = 3$ is a vertical asymptote.

Now pick a number slightly larger than $x = 3$, say 3.1, since the limit is from the right and evaluate $f(x)$. Notice that both the numerator and the denominator are postive.

Thus $\displaystyle\lim_{x\to 3^+} \frac{x-2}{x^2-9} = +\infty$

(h) If you plug in $x = 3$ into $f(x) = \dfrac{x-2}{x^2-9}$, you get the form $\frac{\text{non-zero number}}{0}$. This means that $x = 3$ is a vertical asymptote.

Now pick a number slightly smaller than $x = 3$, say 2.9, since the limit is from the left and evaluate $f(x)$. Notice that the numerator is positive and the denominator is negative.

$\displaystyle\lim_{x\to 3^-} \frac{x-2}{x^2-9} = -\infty$

(i) If you plug in $x = 3$ into $f(x) = \dfrac{x-4}{x^2-9}$, you get the form $\frac{\text{non-zero number}}{0}$. This means that $x = 3$ is a vertical asymptote.

Now pick a number slightly larger than $x = 3$, say 3.1, since the limit is from the right and evaluate $f(x)$. Notice that the numerator is negative and the denominator are postive.

Thus $\displaystyle\lim_{x\to 3^+} \frac{x-4}{x^2-9} = -\infty$

(j) If you plug in $x = 3$ into $f(x) = \dfrac{x-2}{x^2-9}$, you get the form $\frac{\text{non-zero number}}{0}$. This means that $x = 3$ is a vertical asymptote.

Now pick a number slightly smaller than $x = 3$, say 2.9, since the limit is from the left and evaluate $f(x)$. Notice that the numerator is negative and the denominator is negative.

$\displaystyle\lim_{x\to 3^-} \frac{x-4}{x^2-9} = +\infty$