

Challenge Problem:

1) Find the vertical asymptote(s) for the function: $y = \frac{x+3}{x^2-9}$. Justify that your answer is correct.

$$f(x) = \frac{x+3}{x^2-9}$$

The domain of the function is all real numbers except $x = 3$ and $x = -3$.

The following table shows the values of the function as x gets close to -3 .

x	$f(x)$
-3.1	- .1639
-3.01	- .1664
-3.001	- .16664
-3.0001	- .16666

x	$f(x)$
-2.9	- .1695
-2.99	- .16694
-2.999	- .166694
-2.9999	- .166669

Notice that the left table says that

$$\lim_{x \rightarrow -3^-} f(x) = -\frac{1}{6}$$

Notice that the right table says that

$$\lim_{x \rightarrow -3^+} f(x) = -\frac{1}{6}$$

Thus $x = -3$ is not a vertical asymptote.

Now do similar tables for $x=3$.

x	$f(x)$
3.1	10
3.01	100
3.001	1000

x	$f(x)$
2.9	-10
2.99	-100
2.999	-1000

Notice that the left table says that

$$\lim_{x \rightarrow 3^+} f(x) = \infty$$

Notice that the right table says that

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

Thus $x = 3$ is a vertical asymptote.