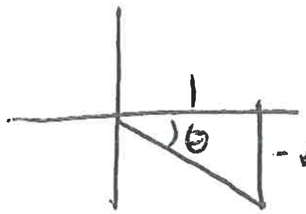
1.5 Additional problems

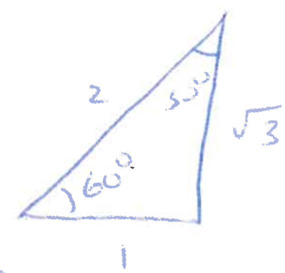
1] a] $\tan^{-1}(-\sqrt{3}) = \theta$

by the definition of arctangent we know $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



So $\tan\theta = -\sqrt{3}$

from the picture on the Right



$\tan 60^\circ = \frac{\sqrt{3}}{1}$ or $\tan\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Thus $\theta = -\frac{\pi}{3}$

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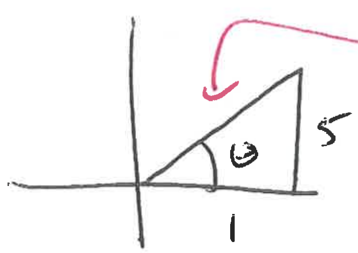
b] $\cos\left(\tan^{-1}(5)\right)$

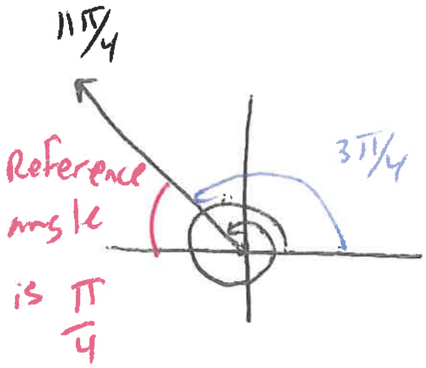
Let $\tan^{-1}(5) = \theta$ or $\tan\theta = 5$ Thus θ in in Quadrant 1



$\sqrt{5^2+1^2} = \sqrt{26}$

$$\cos\left(\tan^{-1}(5)\right) = \cos\theta = \frac{1}{\sqrt{26}}$$

c] $\arcsin\left(\sin \frac{11\pi}{4}\right)$

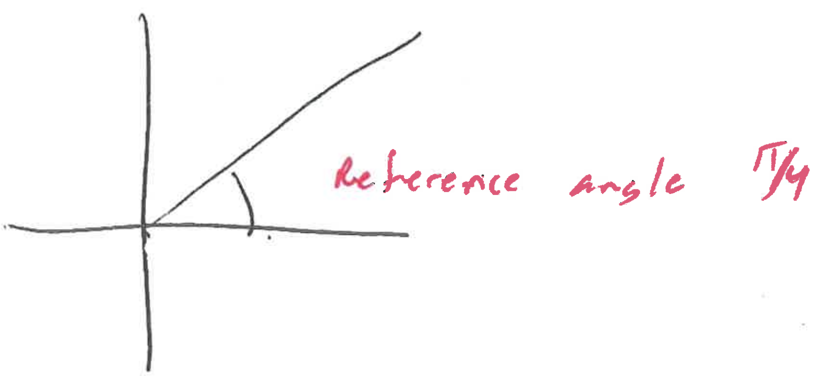


We know arcsine is a value from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

ie Quadrant 1 + 4

$\sin\theta$ is negative for Quadrant 3 (and 4)

Transfer the information to the correct quadrant for the arcsine function.

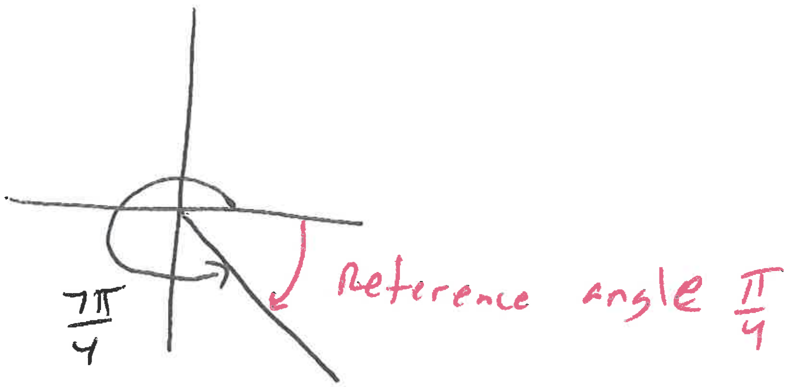


$$\arcsin\left(\sin \frac{11\pi}{4}\right) = \frac{\pi}{4}$$

d) $\arctan\left(\tan \frac{7\pi}{4}\right)$

arctan returns an angle such that

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$\arctan\left(\tan\left(\frac{7\pi}{4}\right)\right) = \frac{-\pi}{4}$

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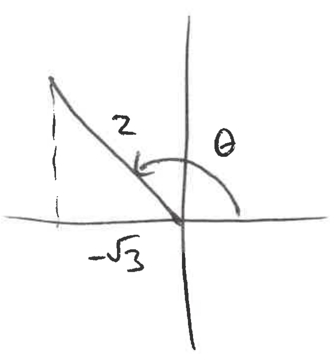
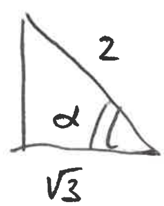
e) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$ we know $0 \le \theta \le \pi$

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\longrightarrow$ says θ is in Quadrant 2





from the Triangle.

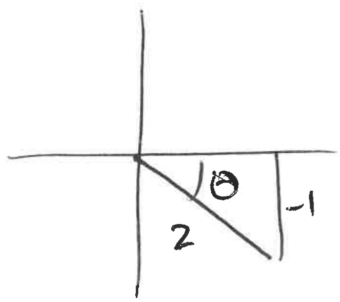
$\cos\alpha = \frac{\sqrt{3}}{2}$

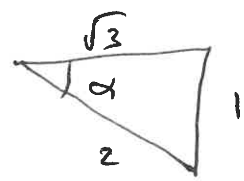
$\alpha = \frac{\pi}{6}$

Thus $\theta = \frac{5\pi}{6}$

f] $\sin^{-1}\left(\frac{-1}{2}\right) = \theta$

$\sin\theta = \frac{-1}{2}$ $\rightarrow$ we Know $\frac{-\pi}{2} \le \theta \le 0$





$\sin\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$

Thus $\underline{\underline{\theta = \frac{-\pi}{6}}}$

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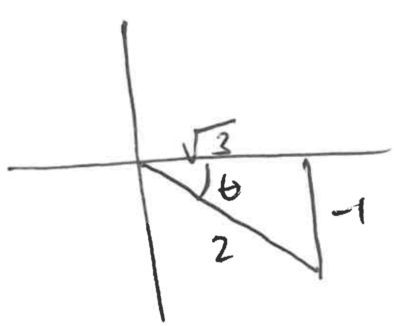
g] we could compute $\sin^{-1}\left(-\frac{1}{2}\right)$ Like a part (f) and then do $\cos\left(\frac{-\pi}{6}\right)$.

<u>2^{nd} method</u> we Know $\sin^{-1}\left(\frac{-1}{2}\right)$ gives an angle between $-\frac{\pi}{2} \le \theta < 0$

so $\sin^{-1}\left(-\frac{1}{2}\right) = \theta$ see the Triangle.

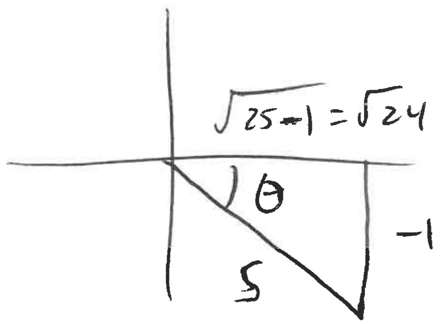


$\underline{\cos(\theta) = \frac{\sqrt{3}}{2}}$

h)

$$\cos\left(\underbrace{\sin^{-1}\left(\tfrac{-1}{5}\right)}_{\text{Quadrant 4}}\right)$$

so $\sin^{-1}\left(\frac{-1}{5}\right) = \theta$



$$\cos\theta = \frac{\sqrt{24}}{5}$$