Sections 5.2: Additioanal Problems Solutions

1. Note: Answers may vary.
   $f(x) = 3x^5 - 6$ and the interval is $[1, 3]$
   $$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(3\left(1 + \frac{2i}{n}\right)^5 - 6\right) = \int_1^3 3x^5 - 6dx$$
2. Note: Answers may vary.
   $f(x) = x^2$ and the interval is $[2, 3]$
   $$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + \frac{i}{n}\right)^2 \frac{1}{n} = \int_2^3 x^2 dx$$
3. base of each rectangle $= \frac{10-2}{4} = 2$
   (a) $2 * f(2) + 2 * f(4) + 2 * f(6) + 2 * f(8) = 11.9013$
   (b) $2 * f(4) + 2 * f(6) + 2 * f(8) + 2 * f(10) = 15.1202$
4. (a) 1170
   (b) 2025
5. Sketch the graph of an increasing function and then draw rectangles that would be used for a left sum.
   Answer: underestimate.
6. overestimate
7. $\int_0^A f(x)\ dx = 10$
8. $\int_A^C g(x)\ dx = 7 - 11 = -4$
9. $\int_C^A f(x)\ dx = -\int_A^C f(x)\ dx = -(-13 + 30) = -17$
10. $\int_C^0 g(x)\ dx = -\int_0^C g(x)\ dx = -[12 + 7 - 11] = -8$
11. $\int_A^B 3f(x)\ dx = 3\int_A^B f(x)\ dx = 3 * (-13) = -39$
12. $\int_0^B [4f(x) + 3g(x)]\ dx = 4\int_0^B f(x)\ dx + 3\int_0^B g(x)\ dx =$
    $4[10 - 13] + 3[12 + 7] = 45$
13. $\int_A^C [3f(x) - 10g(x)]\ dx = 3\int_A^C f(x)\ dx - 10\int_A^C g(x)\ dx =$
    $3[-13 + 30] - 10[7 - 11] = 91$
14. $\int_C^B [-2f(x) + 3g(x)]\ dx = -\left(\int_B^C [-2f(x) + 3g(x)]\ dx\right) =$
    $-\left(-2\int_B^C f(x)\ dx + 3\int_B^C g(x)\ dx\right) =$
    $-(-2 * 30 + 3 * -11) = 93$

15. $\int_0^A [3f(x) + 4g(x)]\ dx = 47$

    $3\int_0^A f(x)\ dx + 4\int_0^A g(x)\ dx = 47$

    $3 * 5 + 4\int_0^A g(x)\ dx = 47$

    $4\int_0^A g(x)\ dx = 32$

    $\int_0^A g(x)\ dx = 8$

16. $2\int_A^B f(x)\ dx - 3\int_A^B g(x)\ dx + 5\int_A^B h(x)\ dx = 150$

    $2 * 12 - 3\int_A^B g(x)\ dx + 5 * 22 = 150$

    $-3\int_A^B g(x)\ dx = 16$

    $\int_A^B g(x)\ dx = \frac{-16}{3}$

17. $7\int_C^D f(x)\ dx + 6\int_C^D g(x)\ dx = 70$

    $6\int_C^D g(x)\ dx = 210$

    $\int_C^D g(x)\ dx = 35$

    Thus $\int_D^C g(x)\ dx = -\int_C^D g(x)\ dx = -35$

18. $\int_0^4 f(x)\ dx = 6$

19. $\int_{-4}^4 f(x)\ dx = \frac{1}{2} * 4 * 2 + 6 = 10$

20. $\int_4^7 f(x)\ dx = -4.5$

21. $\int_1^8 f(x)\ dx = 4 - 7.5 = -3.5$