Section 4.9: Additional Problems Solutions

1. (a) $f'(x) = \dfrac{x^4+20x^2+40}{5x^3} = \dfrac{x^4}{5x^3} + \dfrac{20x^2}{5x^3} + \dfrac{40}{5x^3} = \dfrac{1}{5}x + 4x^{-1} + 8x^{-3}$

   $f(x) = \dfrac{1}{5}\dfrac{x^2}{2} + 4\ln|x| + 8\dfrac{x^{-2}}{-2} = \dfrac{x^2}{10} + 4\ln|x| - \dfrac{4}{x^2} + C$

   (b) $f'(x) = \dfrac{3}{1+x^2} + \dfrac{7}{e^{2x}} + \dfrac{15}{\sqrt{x}} + e^2 = \dfrac{3}{1+x^2} + 7e^{-2x} + 15x^{-1/2} + e^2$

   $f(x) = 3\arctan(x) + \dfrac{7e^{-2x}}{-2} + \dfrac{15x^{1/2}}{1/2} + e^2x + C$

   $f(x) = 3\arctan(x) - \dfrac{7}{2}e^{-2x} + 30\sqrt{x} + e^2x + C$

   (c) $f'(x) = (x\sqrt{x} + \dfrac{7}{x^2} + 3) = x^{1.5} + 7x^{-2} + 3$

   $f(x) = \dfrac{x^{2.5}}{2.5} - 7x^{-1} + 3x + C = \dfrac{2}{5}x^{2.5} - 7x^{-1} + 3x + C$

   (d) $f'(x) = (x^2+5)(x^4+6) = x^6 + 5x^4 + 6x^2 + 30$

   $f(x) = \dfrac{x^7}{7} + x^5 + 2x^3 + 30x + C$

   (e) $f(x) = \dfrac{1}{4}e^{4x} + 2\ln|x| + C$

   (f) $f'(x) = \dfrac{e^{4x}+7e^{2x}}{e^x} = (e^{4x}+7e^{2x}) * e^{-x} = e^{3x} + 7e^x$

   $f(x) = \frac{1}{3}e^{3x} + 7e^x + C$

   (g) $f'(x) = \dfrac{e^{5x}+2xe^{2x}}{e^{2x}} = (e^{5x}+2xe^{2x}) * e^{-2x} = e^{3x} + 2x$

   $f(x) = \frac{1}{3}e^{3x} + x^2 + C$

   (h) $f'(x) = (x^2-3x+1)^2 = x^4 - 6x^3 + 11x^2 - 6x + 1$

   $f(x) = \dfrac{x^5}{5} - \dfrac{6x^4}{4} + \dfrac{11x^3}{3} - 3x^2 + x + C$

   (i) $f'(x) = \sqrt[4]{x^5} + \frac{1}{\sqrt[3]{x^2}} = x^{5/4} + x^{-2/3}$

   $f(x) = \frac{4x^{9/4}}{9} + 3x^{1/3} + C$

2. we know that $\dfrac{d}{dt}\cos(5t) = -5\sin(5t)$ and $\dfrac{d}{dt}\tan(4t) = 4\sec^2(4t)$

   $\mathbf{r}'(t) = \langle 4\sec^2(4t), \sin(5t)\rangle$

   $\mathbf{r}(t) = \left\langle \tan(4t) + c_1, \dfrac{-1}{5}\cos(5t) + c_2 \right\rangle$

   or

   $\mathbf{r}(t) = \left\langle \tan(4t), \dfrac{-1}{5}\cos(5t) \right\rangle + \mathbf{C}$, where $\mathbf{C} = \langle c_1, c_2\rangle$

3. we know $f'(x) = 12x^2 - 6x + 2$ $\qquad$ $f(0) = 1$ and $f(2) = 0$
$f''(x) = 4x^3 - 3x^2 + 2x + C$
$f(x) = x^4 - x^3 + x^2 + Cx + D$

   $f(0) = 1$ gives $1 = 0^4 - 0^3 + 0^2 + C * 0 + D$ or $D = 1$

   $f(2) = 0$ gives $0 = 2^4 - 2^3 + 2^2 + C * 2 + 1$
or $0 = 13 + 2C$ or $C = -7.5$

   Answer: $f(x) = x^4 - x^3 + x^2 - 7.5x + 1$

4. $s(0) = 0$ and $s(b) = 160$ ft. When $t = b$ the car is stopped so we also get $v(b) = 0$.
$a(t) = -40$ ft/sec^2
$v(t) = -40t + C$ since $v(0) = c$ the initial speed and $v(b) = 0$ we find that $0 = -40b + c$ or $b = \frac{c}{40}$ft/sec.

   $s(t) = -20t^2 + Ct + D$ since we set $s(0) = 0$ we find that $0 = 0 + 0 + D$ or $D = 0$.
since $s(b) = 160$ we find that $160 = -20b^2 + Cb$ or
$$160 = -20\left(\frac{c}{40}\right)^2 + c * \left(\frac{c}{40}\right) = -20 * \frac{c^2}{1600} + \frac{c^2}{40} = \frac{-c^2}{80} + \frac{c^2}{40} = \frac{c^2}{80}$$

   $160 = \frac{c^2}{80}$ means that $c^2 = 160 * 80 = 12800$ or $c = 113.14$ft/sec.

   This is equivalent to 77.14mph

5. The first step is to get the speed into the correct units.
60 mph = 88ft/sec.

   (a) $s(0) = 0$, $v(0) = 88$ ft/sec and $a(t) = -22$ft/sec^2.
$v(t) = -22t + C$ and since the initial speed is 88 ft/sec, we see that $C = 88$.
Thus $v(t) = -22t + 88$.
The car will come to a stop when $v(t) = 0$ or $t = 4$sec (assuming no cow on the road).
$s(t) = -11t^2 + 88t + D$ where D is the initial position, $s(0) = 0$ so we get
$s(t) = -11t^2 + 88t$.
Now $s(4) = 176$ft. Since the cow is 160 feet away when the breaks are fully applied, you did hit the cow.

   (b) We want at $t = b$ the car to be at a stop, $v(b) = 0$, 10 feet away from the cow, or $s(b) = 150$ feet.
Let k be the constant deceleration. so $a(t) = -k$
thus $v(t) = -kt + 88$ and $s(t) = \frac{-kt^2}{2} + 88t$ (since the initial position is $s(0) = 0$).

   Since $v(b) = 0$ we get $0 = -kb + 88$ or $k = \frac{88}{b}$

   Since $s(b) = 150$ we get $150 = \frac{-kb^2}{2} + 88b$ or

   $150 = -\frac{88}{b}\frac{b^2}{2} + 88b$ $\qquad$ or $\qquad$ $150 = -44b + 88b = 44b$

   Thus $b = \frac{150}{44} = \frac{75}{22}$ seconds and $k = \frac{88}{b} = 25.81$ft/sec