Section 3.5: Additional Problems Solutions

1. Find the equation of the tangent line at the point $(1, -2)$ for the graph of

   $y^4 + 3y - 4x^3 = 5x + 1$

   Step 1: Take the implicit derivative.

   $4y^3 \frac{dy}{dx} + 3\frac{dy}{dx} - 12x^2 \frac{dx}{dx} = 5\frac{dx}{dx}$

   or

   $4y^3 \frac{dy}{dx} + 3\frac{dy}{dx} - 12x^2 = 5$

   now either solve for $\frac{dy}{dx}$ and then plug in the point or plug in the point and then solve for $\frac{dy}{dx}$.

   $\left(4(-1)^3 + 3\right)\frac{dy}{dx} = 5 + 12(1)^2$

   $-29\frac{dy}{dx} = 17$

   $\frac{dy}{dx} = \frac{-17}{29}$

   Answer: $y + 2 = \frac{-17}{29}(x - 1)$

2. Compute $\frac{dy}{dx}$ for $y^5 - 3x^2y^3 + 5x^4 = 12$

   $5y^4 \frac{dy}{dx} - \left(6xy^3 \frac{dx}{dx} + 9x^2y^2 \frac{dy}{dx}\right) + 20x^3 \frac{dx}{dx} = 0$

   $5y^4 \frac{dy}{dx} - 6xy^3 - 9x^2y^2 \frac{dy}{dx} + 20x^3 = 0$

   $5y^4 \frac{dy}{dx} - 9x^2y^2 \frac{dy}{dx} = 6xy^3 - 20x^3$

   $\left(5y^4 - 9x^2y^2\right)\frac{dy}{dx} = 6xy^3 - 20x^3$

   $\frac{dy}{dx} = \dfrac{6xy^3 - 20x^3}{5y^4 - 9x^2y^2}$

3. Compute $\frac{dy}{dx}$. $\quad \sin(2y)e^{x^2} = \cos(x^3+y^2)$

$$\cos(2y) * 2e^{x^2}\tfrac{dy}{dx} + \sin(2y) * 2xe^{x^2}\tfrac{dx}{dx} = -\sin(x^3+y^2) * \left[3x^2\tfrac{dx}{dx} + 2y\tfrac{dy}{dx}\right]$$

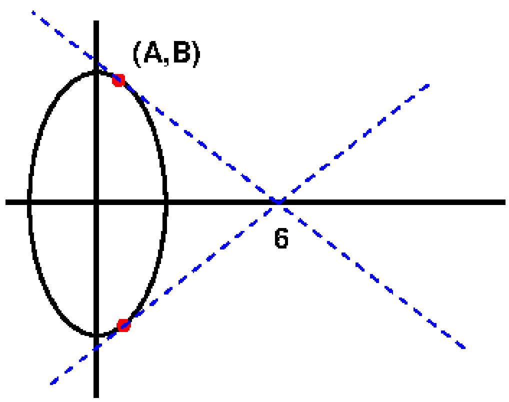
$$\cos(2y) * 2e^{x^2}\tfrac{dy}{dx} + \sin(2y) * 2xe^{x^2} = -3x^2\sin(x^3+y^2) - 2y\sin(x^3+y^2)\tfrac{dy}{dx}$$

$$\cos(2y) * 2e^{x^2}\tfrac{dy}{dx} + 2y\sin(x^3+y^2)\tfrac{dy}{dx} = -\sin(2y) * 2xe^{x^2} - 3x^2\sin(x^3+y^2)$$

$$\left[\cos(2y) * 2e^{x^2} + 2y\sin(x^3+y^2)\right]\tfrac{dy}{dx} = -\sin(2y) * 2xe^{x^2} - 3x^2\sin(x^3+y^2)$$

$$\frac{dy}{dx} = \frac{-\sin(2y) * 2xe^{x^2} - 3x^2\sin(x^3+y^2)}{\cos(2y) * 2e^{x^2} + 2y\sin(x^3+y^2)}$$

4. Here is the picture of the problem. notice that there will be two points of tangency. By the symmetry of the ellipse and the fact that each line goes through the point $(6,0)$, once we get one point, we can change the sign on the y-value and have the second point.



Step 1: Let (A,B) be the point where the upper tangent touches the curve.

The slope of the line through the points (A,B) and $(6,0$ is $m = \dfrac{B}{A-6}$.

Step 2: Find the slope of the tangent line at the point (A,B) using derivatives.

$9x^2 + 4y^2 = 36$

$18x\frac{dx}{dx} + 8y\frac{dy}{dx} = 0$

$18x + 8y\frac{dy}{dx} = 0$

$8y\frac{dy}{dx} = -18x$

$\frac{dy}{dx} = \frac{-18x}{8y} = \frac{-9x}{4y}$

slope at the point is $\dfrac{-9A}{4B}$

Now we get that $\dfrac{-9A}{4B} = \dfrac{B}{A-6}$.

This simplifies to $4B^2 = -9A^2 + 54A$ or $9A^2 + 4B^2 = 54A$

Since the ellipse is $9x^2 + 4y^2 = 36$ and at the point (A, B) this gives $9A^2 + 4B^2 = 36$, we see that $54A = 36$ or $A = \frac{2}{3}$

substituting this value into the equation $9A^2 + 4B^2 = 36$ and solving for B gives.

$B = \pm\sqrt{8}$

Points are $\left(\frac{2}{3}, \sqrt{8}\right)$ and $\left(\frac{2}{3}, -\sqrt{8}\right)$