Section 3.4: Additional Problems Solutions

1. $H'(x) = f'(g(x)) * g'(x)$

   $H'(0) = f'(g(0)) * g'(0) = f'(3) * 5 = -1 * 5$

   Answer: $H'(0) = -5$

2. $J'(x) = 2xf'(x^2)$

   Answer: $J'(-1) = -12$

3. $K'(x) = 3\left(x^2 + g(x)\right)^2 * (2x + g'(x))$

   Answer: $K'(2) = 1512$

4. $J'(x) = 3x^2 * g(3x) + (x^3 + 1) * 3g'(3x)$

   Answer: $J'(1) = 36$

5. $H'(x) = g'(x^2 + f(x)) * (2x + f'(x))$

   Answer: $H'(1) = 64$

6. $y' = 2x(x-3)^4 + x^2 * 4(x-3)^3$
   $y' = 2x(x-3)^3)[(x-3) + 2x]$

   $0 = 2x(x-3)^3(3x-3)$

   Answer: $x = 0$, $x = 1$, $x = 3$

7. $y' = 2(x+1) * (x-3)^3 + (x+1)^2 * 3(x-3)^2$
   $y' = (x+1)(5x-3)(x-3)^2$

   Answer: $x = -1$, $x = 3$, $x = \dfrac{3}{5}$

8. $y' = 7(5-x^2)^6(-2x)(7x+1)^4 + (5-x^2)^7(4*(7x+1)^3 * 7$

   $y' = 7(5-x^2)^6(7x+1)^3\Big((-2)(7x+1) + (5-x^2) * 4\Big)$

   $y' = 14(5-x^2)^6(7x+1)^3\Big((-1)(7x+1) + (5-x^2) * 2\Big)$

   $y' = 14(5-x^2)^6(7x+1)^3\Big(-7x - 1 + 10 - 2x^2\Big)$

   $y' = 14(5-x^2)^6(7x+1)^3\Big(-2x^20 - x + 10\Big)$

   now set equal to zero and solve for x to get

   $x = \pm\sqrt{5}$, $x = \dfrac{-1}{7}$, $x = \dfrac{-10}{9}$, and $x = 1$

9. $f'(x) = 2(x^2+1)^{\frac{1}{2}} + (2x+1)\left(\frac{1}{2}\right)(x^2+1)^{-\frac{1}{2}}(2x)$

10. $f'(x) = (3 + e^x)e^{3x + e^x}$

11. $f'(x) = 4(x^2 + 6x + 1)^3(2x + 6)$

12. $f'(x) = 3^{(x^2+5x+1)}(2x+5)(\ln 3)$

13. $y = e^{(x^2+4x)-(4x+5)} = e^{x^2-5}$

    $y' = 2xe^{x^2-5}$

14. $f'(x) = 1.5(x^3+5x+9)^{\frac{1}{2}}(3x^2+5)$

15. $f'(x) = \frac{1}{3}(x^3+x^{-3})^{-\frac{2}{3}} * (3x^2-3x^{-4})$

16. $f'(x) = 4(3x^5-1)^3(15x^4)*(x^3+2)^3 + 3(x^3+2)^2(3x^2)*(3x^5-1)^4$

17. $f'(x) = 5\left[(x^4-7x^2)^6+4x^3\right]^4 * \left[6(x^4-7x^2)^5(4x^3-14x)+12x^2\right]$

18. $f'(x) = (4x^3+6x)e^{(x^4+3x^2+1)} * (2x^3+7x) + (6x^2+7) * e^{(x^4+3x^2+1)}$

19. $y' = \dfrac{2x\sqrt{x^2+2} - (x^2+1) * \frac{1}{2}(x^2+2)^{-1/2} * 2x}{x^2+2}$

$$y' = \frac{2x\sqrt{x^2+2} - (x^2+1) * (x^2+2)^{-1/2} * x}{x^2+2}$$

now multiply the numerator and denominator each with $\sqrt{x^2+2}$

$$y' = \frac{2x*(x^2+2) - (x^2+1)*x}{(x^2+2)^{3/2}}$$

$$y' = \frac{2x^3+4x-x^3-x}{(x^2+2)^{3/2}} = \frac{x^3+3x}{(x^2+2)^{3/2}}$$

20. $y = \sin(3x+1)$

$y' = 3\cos(3x+1)$

$y'' = -3^2\sin(3x+1)$

$y''' = -3^3\cos(3x+1)$

$y^{(4)} = 3^4\sin(3x+1)$ or $y^{(4)} = 3^4 * y$

Notice: each derivative add another factor of 3 by the chain rule.

Notice after 4 derivatives you are basically back to where you start. so divide 1047 by 4 and see that you get 261 with a remainder of 3 i.e. 3 more derivatives.

$y^{(4)} = -3^{1047}\cos(3x+1)$