Section 3.3: Additional Problems Solutions

1. by the quotient rule: $y' = \dfrac{\sec(x) * \sec^2(x) - (\tan(x) - 1) * \sec(x)\tan(x)}{\sec^2(x)}$

   If you do a little bit of algebra before taking the derivative.

   $$y = \frac{\tan(x) - 1}{\sec(x)} = \frac{\dfrac{\sin(x)}{\cos(x)} - 1}{\dfrac{1}{\cos(x)}} = \left(\frac{\sin(x)}{\cos(x)} - 1\right) * \frac{\cos(x)}{1} = \sin(x) - \cos(x).$$

   $y' = \cos(x) + \sin(x)$

2. $y' = 2\sin(x) + 2x\cos(x)$.

   slope of the tangent line: $y'(\pi/2) = 2$

   Answer: $y - \pi = 2(x - \frac{\pi}{2})$

3. $f'(x) = e^x \cos(x) + e^x * (-\sin(x)) = e^x \cos(x) - e^x \sin(x)$

   or $f'(x) = e^x \Big(\cos(x) - \sin(x) \Big)$

   $f''(x) = e^x \Big(\cos(x) - \sin(x) \Big) + e^x \Big(-\sin(x) - \cos(x) \Big) = -2e^x \sin(x)$

4. This question would not ever be given as an exam question. what it does is give you practice in finding patterns in derivative so that when you take cal 2 life will be a bit easier.

   To find the 35th derivative, we need to look for a pattern. So take enough derivatives that a pattern become apparent.

   $y = x\sin(x)$

   $y' = \sin(x) + x\cos(x)$

   $y'' = \cos(x) + \Big(\cos(x) - x\sin(x) \Big) = 2\cos(x) - x\sin(x)$

   $y''' = -2\sin(x) - \Big(\sin(x) + x\cos(x) \Big) = -3\sin(x) - x\cos(x)$

   $y^{(4)} = -3\cos(x) - \Big(\cos(x) - x\sin(x) \Big) = -4\cos(x) + x\sin(x)$

   $y^{(5)} = 4\sin(x) + \Big(\sin(x) + x\cos(x) \Big) = 5\sin(x) + x\cos(x)$

   $y^{(6)} = 5\cos(x) + \Big(\cos(x) - x\sin(x) \Big) = 6\cos(x) - x\sin(x)$

   $y^{(7)} = -6\sin(x) - \Big(\sin(x) + x\cos(x) \Big) = -7\sin(x) - x\cos(x)$

   first notice that first number is the derivative taken.

   Now lets look at the signs and the trig functions. forget about the first number. Then you should notice that the first derivative looks similar to the 5th derivative. The second derivative looks similar to the 6th derivative. The third derivative looks similar to the 7th derivative. The 4th derivative would look similar to the 8th derivative(if we had done it).

   There is a cycle of 4 with the derivatives.

All of these derivatives look similar:

1st, 5th, 9th, 13th, 17th, 21st, 25th, 29th, 33rd, 37th,....

since the 33rd derivative looks like the 1st derivative, then the 34th derivative would look like the 2nd and the 35 derivative would look like the 3rd derivative.

thus

$y^{(35)} = -35\sin(x) - x\cos(x)$

5. This question would not ever be given as an exam question. what it does is give you practice in finding patterns in derivative so that when you take cal 2 life will be a bit easier.

Lets take lots of derivatives and simplify so we can find a pattern.

$y = \sin(x)e^x$

$y' = \Big(\cos(x) + \sin(x)\Big)e^x$

$y'' = 2\cos(x)e^x$

$y''' = 2\Big(-\sin(x) + \cos(x)\Big)e^x$

$y^{(4)} = -4\sin(x)e^x$ notice that this is also $y^{(4)} = -4y$

$y^{(5)} = -4\Big(\cos(x) + \sin(x)\Big)e^x$ notice that this is also $y^{(5)} = -4y'$

now using the discovery will make the other derivative a bit easier.

$y^{(6)} = -4y''$

$y^{(7)} = -4y'''$

$y^{(8)} = -4y^{(4)} = -4\Big(-4y\Big) = (-4)^2 y$ using the information we know about the 4th derivative.

So every 4 derivatives gets you back to the start with another constant of -4. Notice after 8 derivatives you have a constant of $(-4)^2$ as the starting constant. The power of 2 is for the two cycles.

To figure out how many cycles we need divide 41 by 4. we get 10 cycles with a remainder of 1. the remainder means one more derivative.

$y^{(40)} = (-4)^{10}y$

$y^{(41)} = (-4)^{10}y' = (-4)^{10}\Big(\cos(x) + \sin(x)\Big)e^x$