Section 3.1: Additional Problems Solutions

1. Use any method to find the derivative of $g(x) = |2x + 5|$

   Note: Since we are taking the absolute value of a linear function, we know that $g(x)$ is a continuous function and will have a sharp point at $x = -2.5$.

   As a piecewise defined function we know that $g(x) = \begin{cases} 2x+5 & \text{if } x \geq -2.5 \\ -(2x+5) & \text{if } x < -2.5 \end{cases}$

   Thus $g'(x) = \begin{cases} 2 & \text{if } x \geq -2.5 \\ -2 & \text{if } x < -2.5 \end{cases}$

2. At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

   Solving the line for slope intercept form gives: $y = 3x + 6$.

   Thus this question is asking at what point is the slope of the tangent line equal to 3.

   first $y = x\sqrt{x} = x^{1.5}$

   $y' = 1.5x^{0.5} = 1.5\sqrt{x} = \dfrac{3\sqrt{x}}{2}$

   set $y' = 3$ and solve for x.

   $\dfrac{3\sqrt{x}}{2} = 3$

   $3\sqrt{x} = 6$

   $\sqrt{x} = 2$

   $x = 4$.

   thus the point is $(4, y(4))$ or $(4, 8)$

3. Find the equation of the tangent line at $x = 2$ for $f(x) = \dfrac{x}{x-1}$

   The point that we want the tangent line at is $(2, f(2))$ or $(2, 2)$.

   Now find the slope of the tangent line.

   $f'(x) = \frac{(x-1)*1 - x*(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

   $m_{tan} = f'(2) = \frac{-1}{(2-1)^2} = -1$

   Answer: $y - 2 = -1(x - 2)$

4. Find the value(s) of x where the tangent line to $f(x) = \dfrac{x}{x-1}$ will go through the point $(6, -2)$. Show the work that verifies your answers.

   Notice that the point $(6, -2)$ is not on the graph of the function.

   Let the $x = A$ the value of x where the tangent line at that point will go through $(6, -2)$.

   we will need $f(A)$ and $f'(A)$.

   $f'(x) = \frac{(x-1)*1 - x*(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

   $f'(A) = \dfrac{-1}{(A-1)^2}$ and $f(A) = \dfrac{A}{A-1}$

   Now $y - f(A) = f'(A)(x - A)$ is the equation of the tangent line.

   $y - \dfrac{A}{A-1} = \dfrac{-1}{(A-1)^2}(x - A)$

   now plug in the point for the x and y.

   $-2 - \dfrac{A}{A-1} = \dfrac{-1}{(A-1)^2}(6 - A)$

   now multiply both sides of the equation by $(A-1)^2$ to siplify the equation.

   $-2(A-1)^2 - A(A-1) = -(6 - A)$

   $-2(A^2 - 2A + 1) - A^2 + A = -6 + A$

   $-2A^2 + 4A - 2 - A^2 + A = -6 + A$

   $-3A^2 + 4A + 4 = 0$ or $3A^2 - 4A - 4 = 0$

   $(3A + 2)(A - 2) = 0$

   Answer: 2 and $\dfrac{-2}{3}$

5. Since $y = 2x + 3$ is the tangent to the curve that means that at some value $x = A$, we know the slope of the tangent line is 2. i.e. $f'(A) = 2$.

   From the equation we know $f'(x) = y' = 2cx$. Thus we get that at $x = A$ the following.

   $2 = 2cA$ or $A = \frac{1}{c}$.

   Since the tangent line and the function share the point at $x = A$, i.e. same $y-$values, we see that

   $2A + 3 = cA^2$.

   now replace A with $A = \frac{1}{c}$ to get $\frac{2}{c} + 3 = c * \frac{1}{c^2}$ or $\frac{2}{c} + 3 = \frac{1}{c}$

   multiplying the equation by c, since we know c is not zero, we get $2 + 3c = 1$ or $3c = -1$ or $c = \frac{-1}{3}$

6. For $x \neq 2$ we know $f'(x) = \begin{cases} 2x & \text{if } x < 2 \\ m & \text{if } x > 2 \end{cases}$

   For $f(x)$ to be differentiable at $x = 2$ we need $f(x)$ to be continuous and smooth at $x = 2$. This means we need $f'(x)$ to be continuous at $x = 2$.

   <u>$f'(x)$ continuous needs</u>

   $\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$ or $4 = m$

   <u>$f(x)$ continuous needs</u>

   $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ or $4 = 2m + b$

   This means that $m = 4$ and $b = -4$

7. $y = x^5 + 3x^3 + 7x^2$

   $y' = 5x^4 + 9x^2 + 14x$

8. $y = x^{3.5} + 4x^{1.5} + x^{0.5}$

   $y' = 3.5x^{2.5} + 6x^{0.5} + 0.5x^{-0.5}$

9. $y = x^{3/5} + x^{2/3} + 7^2$

   $y' = \frac{3}{5}x^{-2/5} + \frac{2}{3}x^{-1/3}$

10. $y = 14x^{-10/7} + \pi^4 + x^{1.8}$

    $y' = -20x^{-17/7} + 1.8x^{0.8}$

11. $y' = 3x^2 - 10x + 6$
$6 = 3x^2 - 10x + 6$
$0 = 3x^2 - 10x$
$0 = x(3x - 10)$
Answer: $x = 0$, $x = \frac{10}{3}$

12. $x = -4$, $x = \frac{2}{3}$

13. $x = 8$, $x = -4$

14. $f'(x) = 3x^2 + 2Bx$ and we want $f'(2) = 30$

$30 = 3(2)^2 + 2B(2)$
$30 = 12 + 4B$

Answer: $B = 4.5$

15. $B = 8$

16. The equation of the tangent line at $x = 3$ is
$y - 12 = 6(x - 3)$ or
$y = 6x - 6$
now set y to zero and solve for x.

Answer: $x = 1$

17. $x = 1.5$