Appendix K.1: Additional Problems

1. $\mathbf{v}(t) = \mathbf{r}'(t) = \left\langle \frac{t}{\sqrt{t^2+7}}, 1 \right\rangle$

   $\mathbf{v}(3) = \left\langle \frac{3}{4}, 1 \right\rangle$

   speed $= |\mathbf{v}(3)| = \sqrt{\frac{9}{16} + 1} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

2. First find the intersection point.

   $\mathbf{r_1}(t) = \mathbf{r_2}(s)$ means that $1 - t = s - 2$ and $3 + t^2 = s^2$

   Solve the left equation to get $3 - t = s$ and substitute into the right equation.

   $3 + t^2 = (3 - t)^2$

   $3 + t^2 = 9 - 6t + t^2$

   $3 = 9 - 6t$

   $6t = 6$ or $t = 1$.

   This gives $s = 2$

   Intersection point is $(0, 4)$.

   Now find the two tangent vectors.

   $\mathbf{r_1'}(t) = \langle -1, 2t \rangle$ and $\mathbf{r_2'}(s) = \langle 1, 2s \rangle$

   $\mathbf{r_1'}(1) = \langle -1, 2 \rangle$ and $\mathbf{r_2'}(2) = \langle 1, 4 \rangle$

   Now use the dot product to find θ.

   $\mathbf{r_1'}(1) \cdot \mathbf{r_2'}(2) = |\mathbf{r_1'}(1)||\mathbf{r_2'}(2)| \cos\theta$

   $7 = \sqrt{5}\sqrt{17}\cos\theta$

   $\theta = \arccos\left(\frac{7}{\sqrt{5}\sqrt{17}}\right) = 40.6^o$