

## Chapter 10: The Term Structure of Interest Rates

### Section 10.5: Forward Rates

A **forward rate** is an expected spot rate which will come into play in the future.

Consider the following: A firm needs to borrow money for two years. The one-year spot rate is 7% and the two-year spot rate is 8%.

The firm has two options: (a) borrow all the money at the two-year spot rate or (b) borrow for one year at the one-year spot rate and then borrow for the second year at the one-year spot rate in effect a year later. The second one-year spot rate is called a **forward rate**.

A set of spot rates will imply a set of forward rates.

Unless told otherwise, forward rates are quoted as annual effective rates.

#### Notation:

- $s_t$  is the spot rate from time 0 (year 0) to time  $t$  (year  $t$ ).
- $f_t$  is the one year forward rate from year  $t$  to year  $t + 1$ .  
i.e.  $f_2$  means, starting 2 years from now the effective rate of interest for one year will be  $f_2$ .

Interpret  $f_0$ .

Example: Given the following spot rates, find all one-year forward rates that can be determined from this information.

term	1 year	2 year	3 year	4 year
Spot rate, $s_t$	6%	6.25%	7%	7.5%

In general, an  $n$ -year spot rate can be expressed in terms of a set of  $n$  one-year forward rates.

Example: The following table has the prices of \$1000 par value bonds with 10% annual coupons.

term	1 year	2 year	3 year
price	1028.04	1036.53	1034.47

Find the forward rates for  $t = 0, 1, 2$  that are implied by these bond prices.

Forward rate over m-years

The m-year forward rate (annual effective) which applies over the period from time  $t$  to time  $t + m$  is denoted by  ${}_m f_t$  or  $f_{t,t+m}$

Example: Given the following spot rates, compute the forward rate that is applicable for 3 years starting 2 years from now.

term	1 year	2 year	3 year	4 year	5 year
Spot rate, $s_t$	6%	6.25%	7%	7.5%	8%

Example: Consider the forward rates given below.

t	0	1	2	3	4	5	6
$f_t$	2%	4%	5%	7%	8%	9%	3%

Compute  ${}_4f_2$ .