Section 16.9: Divergence Theorem

Divergence Theorem Let E be a simple solid region whose boundary surface has positive (outward) orientation. Let $\mathbf{F}$ be a vector field whose component functions have continuous partial derivatives on an open region that contains E. Then

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F} dV$$

Example: Let E be the solid bounded by $x^2 + y^2 = 25$, $z = 0$, and $z = 10$. Find the flux of the vector field $\mathbf{F} = \langle 1 + x, 2 + 3y, 2z + 5 \rangle$ over the boundary of the solid. Use positive orientation.

Example: Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \left\langle xy,\ y^2 + e^{xz^2},\ \sin(xy) \right\rangle$ and S is the surface (with positive orientation) of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the planes $z = 0$, $y = 0$, and $y + z = 2$.

Example: Let S be the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$. Let $\mathbf{F} = \left\langle xz,\ yz,\ 3z^2 \right\rangle$. Use positive orientation.

Evaluate $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$ where S is the boundary of the solid.

Example: Let S_1 be the surface of the paraboloid $z = x^2 + y^2$ for $0 \leq z \leq 1$ with downward orientation. Let $\mathbf{F} = \langle xz, yz, 3z^2 \rangle$.

Compute $\displaystyle\iint_{S_1} \mathbf{F} \cdot d\mathbf{S_1}$