Section 15.7 : Triple Integrals in Cylindrical Coordinates

Cylindrical Coordinates:
A Cartesian point (x, y, z) is represented by (r, θ, z) in the Cylindrical Coordinate System. Where (r, θ) represent the polar coordinates for the point (x, y) and z is the distance above or below the $xy-$plane.

$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z \qquad r^2 = x^2 + y^2 \qquad \tan\theta = \dfrac{y}{x}$

Note: Cylindrical coordinates are useful in problems that involve symmetry about the $z-$axis.

Example: Find the cylindrical coordinates for the point $(-1, \sqrt{3}, 2)$

Example: Write the equations in cylindrical coordinates.

A) $z = 12 - 4x^2 - 4y^2$

B) $z = \sqrt{3x^2 + 3y^2}$

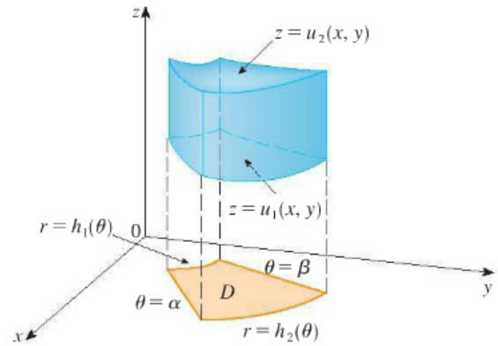
Triple Integrals in Cylindrical Coordinates

Suppose that E is a solid whose image D on the $xy-$plane can be described in polar coordinates.

$E = \{(x, y, z)|(x, y) \in D, u_1(x, y) \leq z \leq u_2(x, y)\}$

and $D = \{(r, \theta)|a \leq \theta \leq b, h1(\theta) \leq r \leq h2(\theta)\}$

If $f(x, y, z)$ is continuous over the solid E,
$g_1(r, \theta) = u_1(r\cos\theta, r\sin\theta)$, and
$g_2(r, \theta) = u_2(r\cos\theta, r\sin\theta)$ then



$$\iiint\limits_E f(x, y, z)dV = \int\limits_{\theta=a}^{b} \int\limits_{r=h_1(\theta)}^{h_2(\theta)} \int\limits_{z=g_1(r,\theta)}^{g_2(r,\theta)} f(r\cos\theta, r\sin\theta, z)\ r\ dzdrd\theta$$

Example A solid lies with in the cylinder $x^2 + y^2 = 4$, below the plane $z = 8$, and above the paraboloid $z = 4 - x^2 - y^2$. The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

Example: Evaluate $\displaystyle\int_{x=0}^{2}\int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{z=\sqrt{x^2+y^2}}^{2}(x^2+y^2)\;dzdydx$