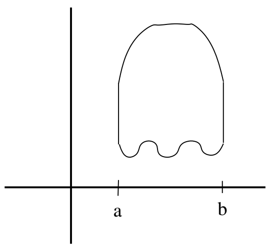
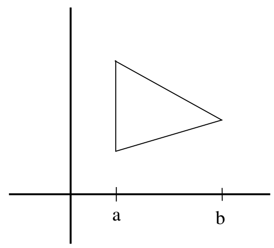
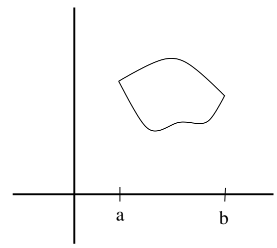
Section 15.2: Double Integrals over General Regions

Definition: A plane region D is said to be of **Type I** if it lies between two continuous functions of x, that is

$D = \{(x, y) |\ a \leq x \leq b,\ B(x) \leq y \leq T(x)\}$





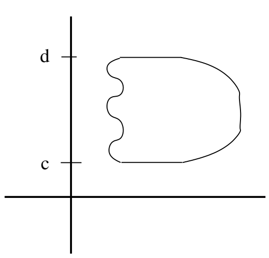


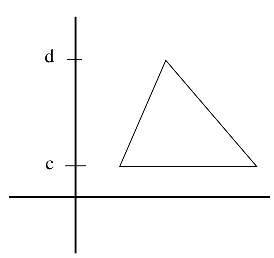
Theorem: If f is continuous on a type I region D such that
$D = \{(x, y) |\ a \leq x \leq b, B(x) \leq y \leq T(x)\}$ then

$$\iint_D f(x,y)dA = \int_{x=a}^{b} \int_{y=B(x)}^{T(x)} f(x,y)\ dydx$$

Definition: A plane region D is said to be of **Type II** if it lies between two continuous functions of y, that is

$D = \{(x, y) |\ c \leq y \leq d,\ L(y) \leq x \leq R(y)\}$





Theorem: If f is continuous on a type II region D such that
$D = \{(x, y) |\ c \leq y \leq d, L(y) \leq x \leq R(y)\}$then

$$\iint_D f(x,y)dA = \int_{y=c}^{d} \int_{x=L(y)}^{R(y)} f(x,y)\ dxdy$$

Example: If D is the region bounded by $y = 4x$ and $y = x^2$ evaluate

$$\iint_D x + y \, dA$$

Example: Set up the double integral that would evaluate the function $f(x, y) = x + y$ over the region bounded by the lines $y = 6x$, $y = 2x$, and $y = -2x + 16$

Example: Evaluate the integral by changing the order of integration $\int_0^1 \int_{2x}^2 \cos(y^2)\ dydx$

Example Find the volume of the solid bounded by the cylinder $x^2 + z^2 = 9$, the planes $x = 0$, $y = 0$, $z = 0$, $x + 2y = 2$ in the first octant.

Example: Setup the integral(s) that would give the volumn of the solid (a tetrahedron) bounded by the planes $x = 0$, $z = 0$, $x = 3y$ and $x + y + z = 4$

Example: Evaluate $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dx dy$

Properties of Double Integrals:

- If a is a real number then $\displaystyle\iint_D (af(x,y) + g(x,y))dA = a\iint_D f(x,y)dA + \iint_D g(x,y)dA$
- If $D = D_1 \cup D_2$, where D_1 and D_2 do not overlap except perhaps on their boundaries then $\displaystyle\iint_D f(x,y)dA = \iint_{D_1} f(x,y)dA + \iint_{D_2} f(x,y)dA$
- $\displaystyle\iint_D 1\ dA = A(D) =$ the area of region D.