

Section 14.1 Functions of Several Variables

Definition: A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $z = f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is $\{f(x, y) | (x, y) \in D\}$.

Definition: If f is a function of two variables with domain D , the **graph** of f is the set $\{(x, y, z) \in \mathbb{R}^3 \mid z = f(x, y), (x, y) \in D\}$.

Example: Find the domain and sketch the graph of the function. What is the range?

(a) $f(x, y) = 10 - x - 2y$

(b) $f(x, y) = \sqrt{16 - x^2 - y^2}$

Example: Find and sketch the domain.

(a) $z = \frac{\sqrt{y - x^2}}{1 - x^2}$

Definition: The **level (contour) curves** of a function of two variable are the curves with the equations $f(x, y) = k$, where k is a constant in the range of f .

Example: Sketch level curves for $f(x, y) = x^2 + y^2$

Example: Sketch a contour graph for $z = \ln(y^2 - x)$

Example: Find the domain of $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 - 25}}$

Example: Sketch the **level surfaces** for $f(x, y, z) = x^2 + y^2 - z$