Section 13.2: Derivatives and Integrals of Vector Functions

Theorem Let $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$, where f, g, and h are differentiable functions, then $\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t)\rangle = f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}$

Note: If $\mathbf{r}(t)$ is a **position function** of a particle at time t, then the **velocity function** is $\mathbf{r}'(t) = \mathbf{v}(t)$ and the **acceleration function** is $\mathbf{r}''(t) = \mathbf{v}'(t) = \mathbf{a}(t)$.

Definition: The **unit tangent vector** at t is defined to be $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

Theorem Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Then.

$$\frac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t) \qquad \frac{d}{dt}[\mathbf{u}(t)\cdot\mathbf{v}(t)] = \mathbf{u}'(t)\cdot\mathbf{v}(t) + \mathbf{u}(t)\cdot\mathbf{v}'(t)$$

$$\frac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t) \qquad \frac{d}{dt}[\mathbf{u}(t)\times\mathbf{v}(t)] = \mathbf{u}'(t)\times\mathbf{v}(t) + \mathbf{u}(t)\times\mathbf{v}'(t)$$

$$\frac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t) \qquad \frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t)) \quad \text{chain rule}$$

Example: Given $\mathbf{r}(t) = \langle 3t,\ e^{2t-4},\ \sin(t\pi)\rangle$.

(a) Find a tangent vector to the curve at $t = 0$.

(b) Find $\mathbf{T}(0)$.

(c) Find a tangent line to the curve at the point $(6, 1, 0)$

Example: Show that if $|\mathbf{r}(t)| = c$ (a constant), then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$

Example: Given $\mathbf{r}(t) = \langle 3t,\ e^{2t-4},\ \sin(t\pi)\rangle$. compute $\int \mathbf{r}(t)\ \ dt$