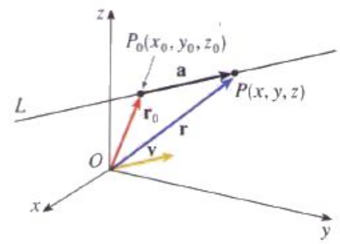
Section 12.5: Equations of Lines and Planes

Definition: The vector equation of a line is found by the formula

$\mathbf{r} = \mathbf{r_0} + t\mathbf{v}$

where $\mathbf{r_0}$ is a vector representation of a point on the line, $\mathbf{v}$ is a directional vector of the line (i.e. a vector that is parallel to the line), and $t \in \Re$.



Example: Find the vector equation and the parametric equations of a line though the point $(1, 2, 3)$ where the line is parallel to the vector $\mathbf{v} =< 2, 5, 10 >$.

Example: Find the vector equation of the line through the points $(3, 5, 5)$ and $(2, 1, -5)$. Also give the parametric equations of this line. Where does the line intersect the xy-plane?

Example: Is the point $(7, 10, 17)$ on the line $\mathbf{r} =< 1 + 3t,\ 2 + 4t,\ 3 + 7t >$?

Symmetric equations of a line: If a, b, c $\neq 0$ and line L goes through the point (x_0, y_0, z_0) with directional vector $\langle a, b, c\rangle$ then

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

If, for example, $a = 0$ then the symmetric equations have the form:

$$x = x_0, \ \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Example: Find the symmetric equations of the line through the point $(5, 8, -2)$ and parallel to the line

$x = 2 + 4t$
$y = 3 + 2t$
$z = 1 + 6t$

Definition: Skew lines are lines that are not parallel and do not intersect.

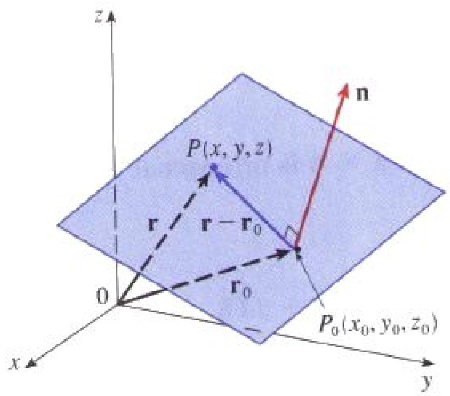
Example: Are these lines parallel, skew, or intersecting? If intersecting, find the point of intersection.

$$L_1: \quad \frac{x + 2}{3} = \frac{y - 5}{-4} = \frac{1 - z}{2}$$

and

$$L_2: \quad x = 1 - t, \quad y = 3 + 2t, \quad z = -12 - 3t$$

A **plane** is determined by a point $P_0(x_0, y_0, z_0)$ and a vector $\mathbf{n} =< a, b, c >$ that is orthogonal to the plane. The vector $\mathbf{n}$ is called a normal vector.



Vector equation of the plane:

Scalar equation of the plane:

Example: Find an equation of the plane through the point $(1, 2, 3)$ and is orthogonal to the vector $\langle 3, 4, 7\rangle$

Example: Find an equation of the plane through the points $A(1, 1, 3)$, $B(-1, 3, 2)$, and $C(1, -1, 2)$.

Example Find an equation of the plane through the point $(1, 2, 3)$ and contains the line $x = 2 + 4t$, $y = 1 + 5t$, $z = -1 + 3t$

Example: You are given two different lines. Does there exist a plane that contains the given lines? If not, what conditions are needed so that there is a plane that contains the given lines?

Definition: Two planes are parallel if their normal vectors are parallel.

Definition: Two planes are perpendicular(orthogonal) if their normal vectors are perpendicular.

Definition: The angle between two non-parallel planes is the acute angle between the normal vectors.

Example: Determine if the pairs of are parallel, orthogonal, or neither?

$P_1: \quad 4x + 2y - 8z = 15$

$P_2: \quad 2x + y - 4z = 12$

$P_3: \quad 3x + 2y + 2z = 10$

Example: Find an equation of the line of intersection, L, of these two planes.

$x - y + 3z = 0$

$x + y + 4z = 2$

The distance between a point P (x, y, z) to the plane $ax + by + cz + d = 0$ is

Example: Find the distance between the point $(3, -2, 7)$ and the plane $4x - 6y + z = 5$