Section 12.3: Vectors

Definition: The **dot product** of two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ is the number

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

where θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, $0 \leq \theta \leq \pi$. If either $\mathbf{a}$ or $\mathbf{b}$ is $\mathbf{0}$, then $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$.

The **dot product** of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Definition: Two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal(perpendicular) if and only if $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$, i.e. the angle between them is $\pi/2$.

Example: Find the following using these vectors: $\mathbf{a} = \langle -1, -2, -3 \rangle$, $\mathbf{b} = \langle -10, 2, 1 \rangle$, and $\mathbf{c} = \langle 2, 8, -6 \rangle$.

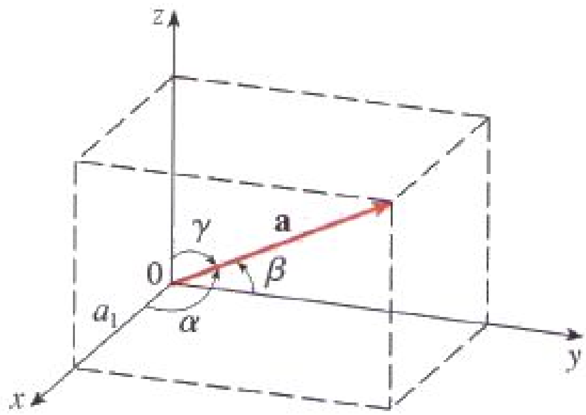
A) $\mathbf{a} \cdot \mathbf{b} =$

B) $\mathbf{a} \cdot \mathbf{c} =$

C) Find the angle between $\mathbf{a}$ and $\mathbf{b}$.

Example: If $|\mathbf{a}| = 1$ and $|\mathbf{b}| = 2$, what is the maximum for $\mathbf{a} \cdot \mathbf{b}$? What does this say about the vectors?

Directional angles/and Direction Cosines

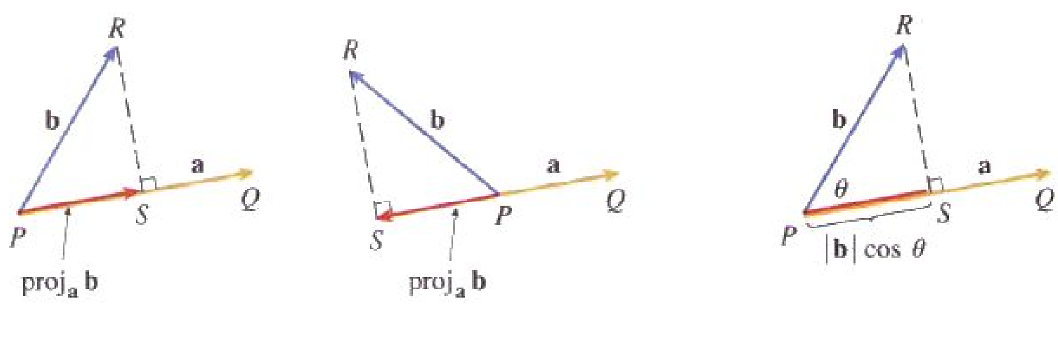


Example: Find the direction angles for $\mathbf{a} = \langle 1, 0, 5 \rangle$

Projections

Scalar projection of **b** onto **a**: $\text{comp}_{\mathbf{a}}\ \mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of **b** onto **a**: $\text{proj}_{\mathbf{a}}\ \mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$



Vector projections

Scalar projection

Example: Find the vector and scalar projections of $\mathbf{m} = \langle 2, 1, 5 \rangle$ onto $\mathbf{n} = \langle 1, 2, 3 \rangle$