

## Section 12.2: Vectors

The term **vector** is used to represent a quantity that has both magnitude and direction. we denote a vector by putting a letter in boldface(  $\mathbf{a}$ ) or by putting an arrow above the letter ( $\vec{a}$ ). The **zero vector**, denoted  $\mathbf{0}$ , has length zero and is the only vector that does not have a specific direction.

**Definition:**A two dimensional vector has the form  $\mathbf{a} = \langle a_1, a_2 \rangle$  and a three dimensional vector has the form  $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ , where  $a_1$ ,  $a_2$ , and  $a_3$  are real numbers and are called the components of the vector.

**Definition:** Given the points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$ , the vector  $\mathbf{a}$  with representation  $\overrightarrow{AB}$  is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

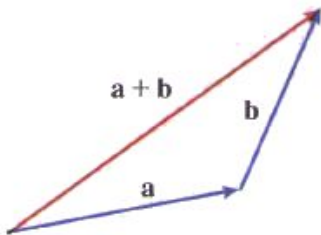
Example: For the points,  $A(1, 2, 8)$  and  $B(4, 7, 2)$ , find  $\overrightarrow{AB}$  and  $\overrightarrow{BA}$ .

**Definition:** Let  $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ ,  $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$  and  $c$  be a scalar, i.e.  $c \in \mathfrak{R}$ .

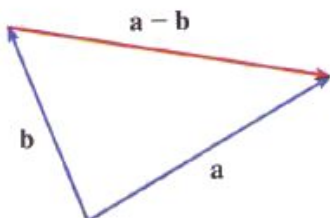
Scalar Multiplication:  $c\mathbf{a} = c \langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$

Length or magnitude of  $\mathbf{a}$  is  $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Vector Addition:  $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

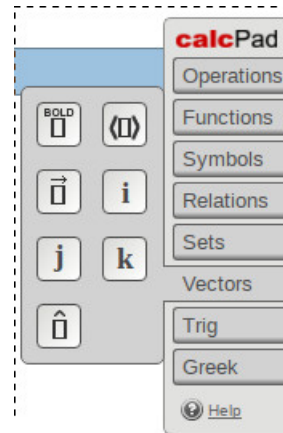


Vector Subtraction:  $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$



**Definition:** Two vectors are parallel if one vector is a scalar multiple of the other. i.e. there exists a  $c \in \mathfrak{R}$  such that  $ca = \mathbf{b}$ .

**Definition:** A vector of length 1 is called a **unit vector**. The vectors  $\mathbf{i} = \langle 1, 0, 0 \rangle$ ,  $\mathbf{j} = \langle 0, 1, 0 \rangle$  and  $\mathbf{k} = \langle 0, 0, 1 \rangle$  are called the standard basis vectors for  $\mathfrak{R}^3$ .



To find a unit vector in the same direction as  $\mathbf{a}$ , divide vector  $\mathbf{a}$  by its magnitude. This process is called normalizing  $\mathbf{a}$ .

Example: Find the following using the vectors:  $\mathbf{a} = \langle 1, 2, 4 \rangle$  and  $\mathbf{c} = \langle 2, -4, 1 \rangle$ .

A)  $3\mathbf{a} - 2\mathbf{c} =$

B) Find a vector of length 3 in the opposite direction of  $\mathbf{a}$ .