Section 11.9: Representations of Functions as Power Series

Geometric Power Series: $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + ... = \frac{1}{1-x}$ converges for $|x| < 1$ with Radius of convergence = 1 and interval of convergence $(-1, 1)$

Example: Find the power series representation of $f(x)$ and the radius and interval of convergence.

A) $\frac{1}{4+x}$

B) $\frac{x^2}{4+x}$

Example: Find the power series representation of $f(x)$ and the radius of convergence.

A) $\dfrac{3x^3}{1-9x^2}$

B) $\dfrac{x}{x^2-3x+2}$

C) $\dfrac{9}{x^4+81}$

Theorem: If the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ has a radius of convergence $R>0$, then the function defined by $f(x)=\sum_{n=0}^{\infty} c_n(x-a)^n$ is differentiable (and therefore continuous) on the interval $(a-R, a+R)$ and

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + ... = \sum_{n=0}^{\infty} c_n(x-a)^n$$

$$f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + ... = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$$

$$\int f(x)dx = C + c_0(x-a) + \frac{c_1(x-a)^2}{2} + \frac{c_2(x-a)^3}{3} + ... = C + \sum_{n=0}^{\infty} \frac{c_n(x-a)^{n+1}}{n+1}$$

The radii of convergence for both $f'(x)$ and $\int f(x)dx$ are both R. The interval of convergence may change.

$f = \sum_{n=0}^{\infty} \frac{x^n}{3^n}$	$g = \sum_{n=0}^{\infty} \frac{x^{n+2}}{3^n}$
$\text{f} = \frac{x^0}{3^0} + \frac{x^1}{3^1} + \frac{x^2}{3^2} + \cdots$	$\text{g} = \frac{x^2}{3^0} + \frac{x^3}{3^1} + \frac{x^4}{3^2} + \cdots$
$\text{f} = 1 + \frac{x}{3} + \frac{x^2}{3^2} + \cdots$	$\text{g} = x^2 + \frac{x^3}{3^1} + \frac{x^4}{3^2} + \cdots$
$f' = \sum_{n=}^{\infty} \frac{nx^{n-1}}{3^n}$	$g' = \sum_{n=}^{\infty} \frac{(n+2)x^{n+1}}{3^n}$

Example: Evaluate this integral by using a power series and find the radius of convergence.

$$\int \frac{9}{x^4 + 81}\, dx$$

Example: Find a power series representation of $f(x)$ and determine the interval and radius of convergence.

$f(x) = \ln(1 + x)$

Example: Find the power series representation of these functions. determine the radius of convergence.

A) $f(x) = \ln(1 - x)$

B) $f(x) = \ln(4 + x^2)$

Example: Find the power series representation of $f(x)$ and determine the radius of convergence.

$f(x) = \arctan(x)$

Example: Find a power series representation of $f(x)$.

$$f(x) = \frac{1}{(1+x)^3}$$

Example: Find a power series representation of $f(x)$.

$$f(x) = \frac{x^3}{(1+2x)^3}$$

Example: Use a series to evaluate this integral.

$$\int \arctan(x^3)\ dx$$