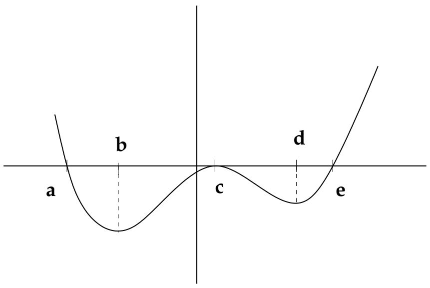
Section 4.1-4.3 Part 1 : Additional Problems

1. Assume that the graph is of $f'(x)$ and the domain of $f(x)$ is all real numbers.



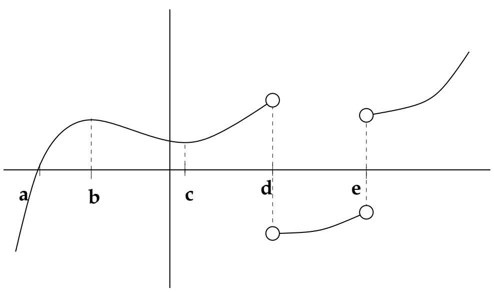
(a) On what intervals is $f(x)$ increasing?

(b) On what intervals is $f(x)$ decreasing?

(c) Give the critical values for $f(x)$ and classify them as local maximum, local minimum, or neither.

(d) Which is larger? $f(b)$ or $f(c)$

2. Assume that the graph is of $f'(x)$ and the domain of $f(x)$ is all real numbers except for $x = e$

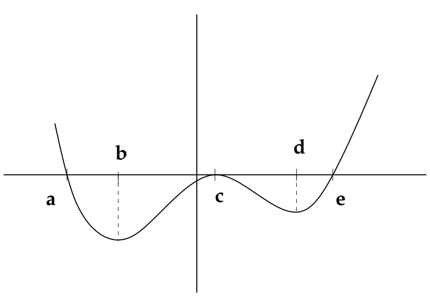


(a) On what intervals is $f(x)$ increasing?

(b) On what intervals is $f(x)$ decreasing?

(c) Give the critical values for $f(x)$ and classify them as local maximum, local minimum, or neither.

3. Assume that the graph is of $f'(x)$ and the domain of $f(x)$ is all real numbers.



(a) On what intervals is $f(x)$ concave up?

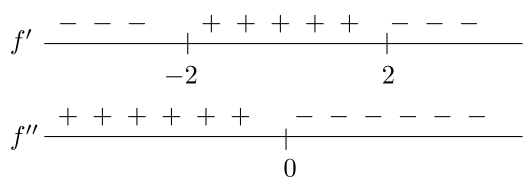
(b) On what intervals is $f(x)$ concave down?

(c) Give the x-values of the inflection points for $f(x)$.

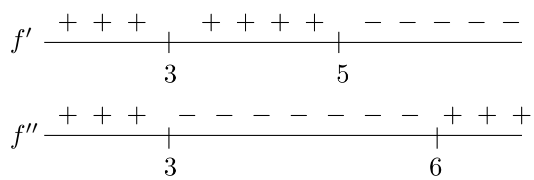
For problems 4-12, sketch a graph of a function that has all of the listed properties.

4. Continuous for all real numbers.
Differentiable for all real numbers.
$f'(-1) = 0$, $f'(1) = 0$
$f(-1) = 4$, $f(1) = 0$.
$f'(x) < 0$ on $(-1, 1)$.
$f'(x) > 0$ on $(-\infty, -1)$ and $(1, \infty)$.
$f''(x) < 0$ on $(-\infty, 0)$.
$f''(x) > 0$ on $(0, \infty)$.

5. Continuous for all real numbers.
Differentiable for all real numbers.
x-intercepts $0, 4$, and -4.
$f'(2) = 0$, $f'(-2) = 0$. $f''(0) = 0$



6. Continuous for all real numbers except $x = 3$
Differentiable for all real numbers except $x = 3$
critical value at $x = 5$
$\lim_{x\to\infty} f(x) = 0$. $\lim_{x\to-\infty} f(x) = 0$



7. Continuous for all real numbers except $x = -2, 0, 2$
Differentiable for all real numbers except $x = -2, 0, 2$
Inflection points at $(-1, 0)$ and $(1, 0)$.
Vertical Asymptote: $x = -2, x = 2$, and $x = 0$.
$\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$
$f'(x) < 0$ on $(-\infty, -2)$ and $(-2, 0)$.
$f'(x) > 0$ on $(0, 2)$ and $(2, \infty)$.
$f''(x) > 0$ on $(-2, -1)$ and $(1, 2)$.
$f''(x) < 0$ on $(-\infty, -2)$, $(-1, 0)$, $(0, 1)$, and $(2, \infty)$.

8. Domain: all real numbers except $x = 2$ and $x = -2$
Continuous for all real numbers except $x = -2, 2$
Not differentiable at $x = -2, 2$
x–intercept: 0
y–intercept: 0
vertical asymptote: $x = -2$ and $x = 2$
horizontal asymptote: none
relative maxima at the point $(4, -4)$
relative minima at the points $(-4, 4)$
inflection point: $(0, 0)$
$f'(x) > 0$ on $(-4, -2)$, $(-2, 2)$, and $(2, 4)$
$f'(x) < 0$ on $(-\infty, -4)$, and $(4, \infty)$
$f''(x) > 0$ on $(-\infty, -2)$ and $(0, 2)$
$f''(x) < 0$ on $(-2, 0)$, and $(2, \infty)$

9. Continuous and differentiable for all real numbers.
$f'(-1) = 0$ and $f'(5) = 0$
$f'(x) > 0$ on $(-1, 5)$ and $(5, \infty)$
$f'(x) < 0$ on $(-\infty, -1)$
$f''(x) > 0$ on $(-\infty, 2)$ and $(5, \infty)$
$f''(x) < 0$ on $(2, 5)$

10. Continuous for all real numbers except $x = 1$ where it has a vertical asymptote.
Differentiable everywhere except at $x = 1$ and $x = 5$
Horizontal asymptote of $y = 0$.
$f'(5) =$DNE and $f(5) = 4$
$f'(x) < 0$ on $(5, \infty)$
$f'(x) > 0$ on $(-\infty, 1)$ and $(1, 5)$
$f''(x) < 0$ on $(1, 5)$
$f''(x) > 0$ on $(-\infty, 1)$ and $(5, \infty)$

11. Continuous for all real numbers.
Differentiable everywhere except at $x = 0$
Horizontal asymptote of $y = 5$.
$f'(2) = 0$ and $f(2) = 1$
$f'(x) < 0$ on $(-\infty, 0)$
$f'(x) > 0$ on $(0, 2)$ and $(2, \infty)$
$f''(x) < 0$ on $(-\infty, 0)$ and $(0, 2)$ and $(4, \infty)$
$f''(x) > 0$ on $(2, 4)$

12. Continuous for all real numbers.
Differentiable everywhere except at $x = 2$
$\lim_{x \to \infty} f(x) = 3$
$f'(6) = 0$ and $f(6) = 6$
$f''(8) = 0$
$f'(x) < 0$ on $(-\infty, 2)$ and $(6, \infty)$
$f'(x) > 0$ on $(2, 6)$
$f''(x) < 0$ on $(2, 8)$
$f''(x) > 0$ on $(-\infty, 2)$ and $(8, \infty)$